Longevity dependence across generations and populations as a risk-mitigation tool in annuity portfolios

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Introduction

Motivation

International expansion is a critical and important driver of *Economic Value* in the Insurance Industry. Some reasons for internationalization are:

- Diversifying risks (e.g. Balancing business cycles)
- Managing costs more efficiently

Geographic distribution of income

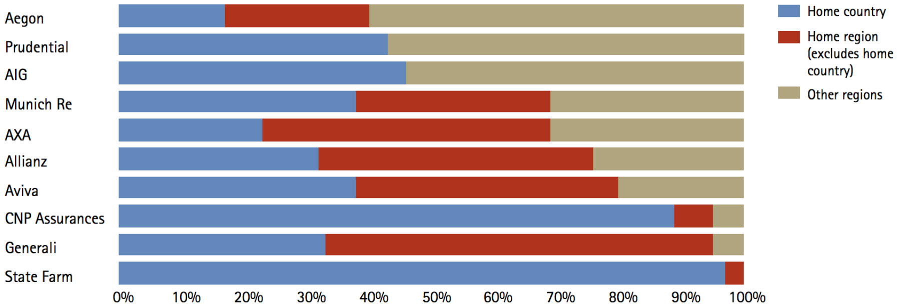

Figure. Geographic distribution of insurance premium income for global top 10 insurers (2008)[1].

[1] Source: Internationalization: a path to high performance for insurers in uncertain times - Accenture Report (2009)

Internationalization of largest Insurers

Table. World's largest Insurers ranked by foreign insurance income in million of dollars[2] (2003).

			Insurance Income		Employment		N. Host
Rank	TNC	Home Country	Foreign	Total	Foreign	Total	Countries
1	Allianz	Germany	75,230	107,180	90,350	173,750	62
2	AXA	France	65,120	84,800	85,490	117,113	46
3	ING	Netherlands	47,990	57,350	80,407	114,344	58
4	Zurich	Switzerland	45,520	48,920	*n.a.*	58,667	46
5	Generali	Italy	38,155	62,500	49,671	60,638	42
6	AIG	US	32,718	70,319	*n.a.*	86,000	92
7	Munich Re	Germany	27,900	50,900	11,060	41,430	36
8	Aviva	UK	26,180	53,480	23,555	56,000	32
9	Swiss Re	Switzerland	25,540	26,940	*n.a.*	7,949	28
10	Winterthur	Switzerland	19,680	27,060	13,865	20,281	16

[2]Source: Outreville, J. F. (2008). Foreign affiliates of the largest insurance groups: Location- specific advantages. Journal of Risk and Insurance 75(2), 463-491.

Number of Life Insurance Undertakings

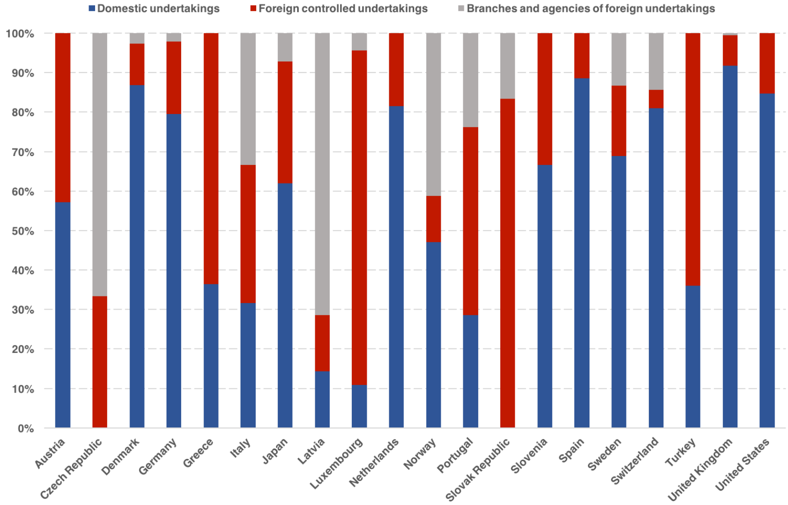

Figure. Number of life insurance undertakings - 2014. Source: OECD.Stat.

Gross Premiums Life Insurance Undertakings

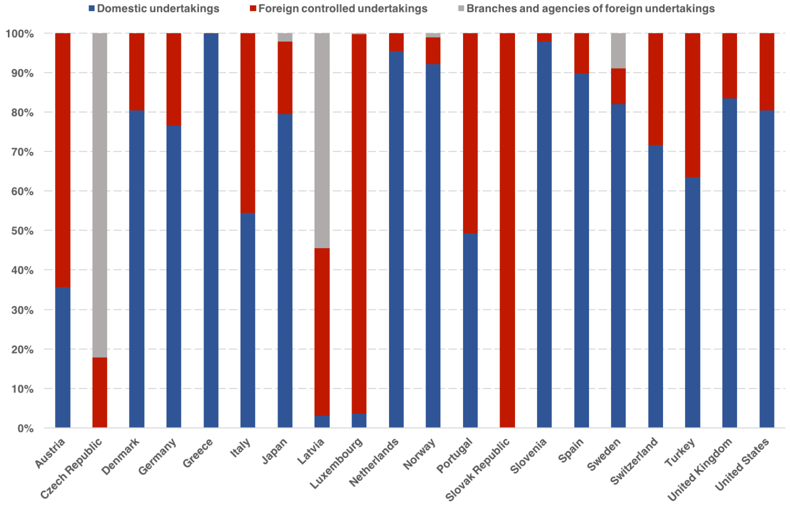

Figure. Gross premiums life insurance undertakings - 2014. Source: OECD.Stat.

Economic Question

- Some recent studies (Outreville (2012), Biener et al. (2015)) analyzed internationalization in the insurance industry, mainly focusing on the **Internationalization-Performance relationship**.
- The main driver of performance linked to internationalization is identified in the **Operational Cost Efficiency**.

We focus on a potential benefit of internationalization in life insurance, arising from the **diversification** gains stemming from **longevity risk** pooling across different populations, that literature has so far neglected.

Economic Question

We consider the case in which the annuity provider, or life insurer, wishes to increase the size of her annuity portfolio and can choose between two possible strategies:

- Sell new contracts to the domestic population,
- Sell new contracts to a foreign population.

Economic Question

- How can we measure the potential added diversification of a foreign population?
- Does geographical diversification in annuity portfolios provide economic benefit (e.g., lower risk margin)?

Preliminary Remarks

Longevity Risk

1. Longevity risk is the risk of unexpected improvements in the survivorship of a given population.

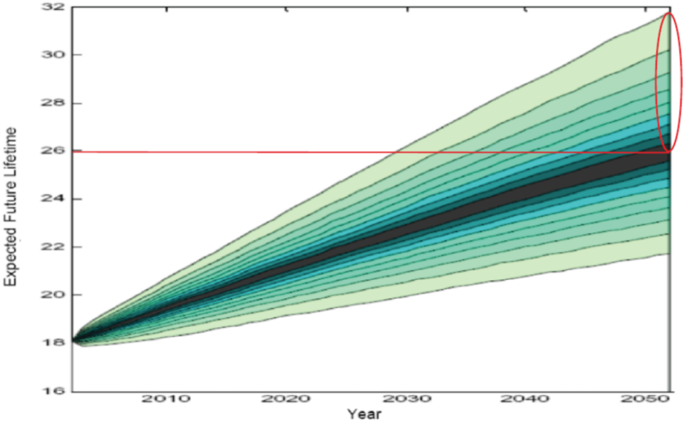

Figure. Source: Dowd K, Blake D, Cairns AJG. Facing Up to Uncertain Life Expectancy: The Longevity Fan Charts. Demography. 2010;47(1):67-78.

Mortality Intensity

2. To model longevity risk we need mortality intensity to be a stochastic quantity:

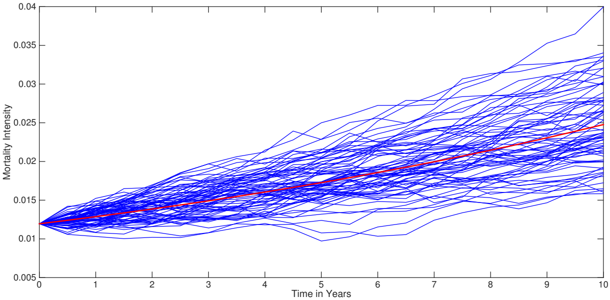

Figure. Mortality Intensity simulations UK 65y males. The red line represents its non-stochastic version.

Survival Probability

3.1 *Current* survival probability at a given horizon is computed as an expectation over the intensity paths in the previous slide.

3.2 *Future* survival probabilities are random because they depend *both* on the future initial value of the intensity (say $\lambda(1)$ at time 1) and the paths of the intensity afterwards.

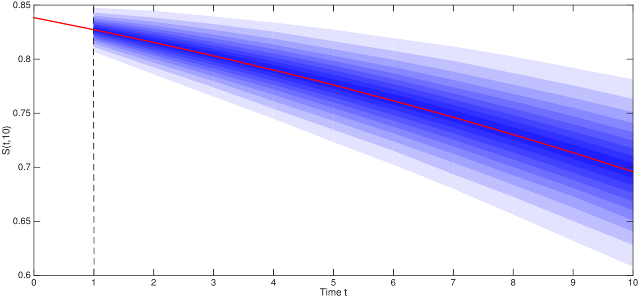

Figure. 10 years Survival Probability simulations UK 65y males. The red line represents its non-stochastic version.

Mortality Intensity of different generations

4. Different generations of the same population have different observed survival probabilities and, therefore, different mortality intensities.

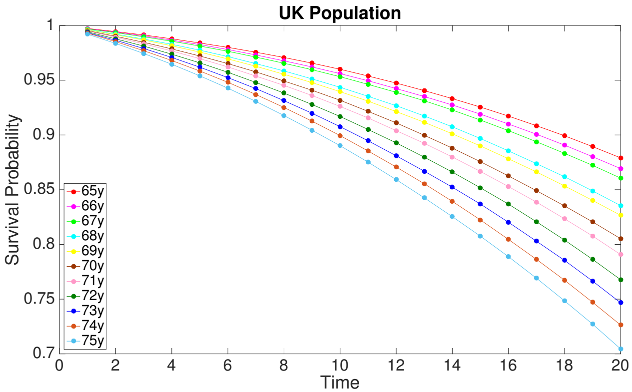

Figure. Observed UK survival probabilities for different generations on 31/12/2012.

- We need to model correlation across generations.

Mortality Intensity of different populations

5. The same generations belonging to different populations have different observed survival probabilities and, therefore, different mortality intensities.

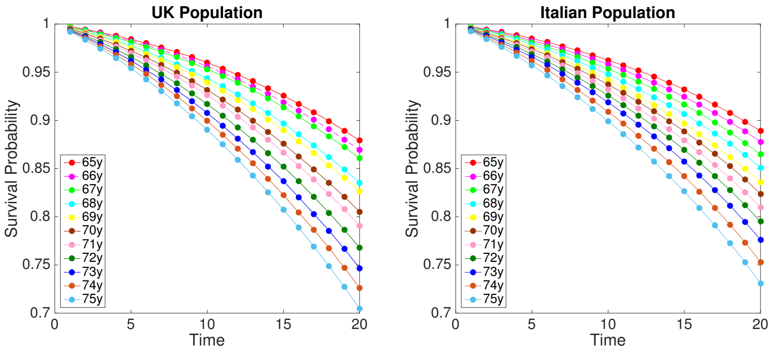

Figure. Observed UK and Italian survival probabilities for different generations on 31/12/2012.

- We need to model dependence across populations.

Theoretical Setup

Aim

First, we provide a mortality model that:

- Accounts for different generations and populations parsimoniously,
- Permits to compute the similarity between the longevity of different populations explicitly,
- Allows to compute correlations between populations,
- Is analytically tractable,
- Can be coupled with one of the best known models for interest rate risk and still gives analytic solutions,
- Allows the computation of sensitivities and hedging ratios (greeks) explicitly.

Mortality Model

Domestic population:

- If we have a single generation x (see De Rosa et al. (2016), SAJ)⇒ its mortality intensity is:

$$d\lambda_x^d(t) = (a + b\lambda_x^d(t))dt + \sigma\sqrt{\lambda_x^d(t)}dW_x(t), \quad (1)$$

Gompertz Mortality

- If we have multiple generations x_i , for $i = 1, \ldots, N$,⇒

$$d\lambda_{x_i}^d = (a_i + b_i\lambda_{x_i}^d)dt + \sigma_i\sqrt{\lambda_{x_i}^d}dW_i, \quad (2)$$

where $a_i, b_i, \sigma_i, \lambda_{x_i}^d(0) \in \mathbb{R}^{++}$ are strictly positive real constants and the W_i's are instantaneously correlated standard Brownian Motions, i.e. $dW_i dW_j = \rho_{ij} dt$ with $i, j \in \{1, \ldots, N\}$.

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Mortality Model

Foreign population:

- If we have a single generation $x \Rightarrow$

$$\lambda_x^f = \delta\lambda_x^d + (1-\delta)\lambda_x', \tag{3}$$

where

$$d\lambda_x' = (a' + b'\lambda_x')dt + \sigma'\sqrt{\lambda_x'}dW_x', \tag{4}$$

Delta

- If we have multiple generations x_i , for $i = 1, \ldots, N, \Rightarrow$

$$\lambda_{x_i}^f = \delta_i\lambda_{x_i}^d + (1-\delta_i)\lambda_{x_i}', \tag{5}$$

where

$$d\lambda_{x_i}' = (a(x_i) + b(x_i)\lambda_{x_i}')dt + \sigma(x_i)\sqrt{\lambda_{x_i}'}dW',$$

  - $\delta_i \in [0, 1]$,
  - $a(x_i)$, $b(x_i)$, and $\sigma(x_i)$ are deterministic functions of x_i,
  - $W' \perp W_i$ for each $i = 1, \ldots, N$.

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  - $W' \perp W_i$ for each $i = 1, \ldots, N$.

Idiosyncratic component specification

Specification 1: The most simple specification for λ'_{x_i} consists in assuming a, b, σ constant, that we define as $a', b', \sigma' > 0$.

- The mortality intensity of each generation belonging the foreign population has the same Idiosyncratic Factor λ', but a different sensitivity δ_i.
- Since $b' > 0$, λ' is a non-mean reverting process, which is consistent with the empirical evidence on cohort-based intensities (Luciano and Vigna (2005)).

Specification 2

Correlation between populations

Assuming $0 \leq u \leq t$, the conditional correlation between $\lambda^d_{x_i}(t)$ and $\lambda^f_{x_i}(t)$ is given by:

$$Corr_u\left[\lambda^d_{x_i}(t), \lambda^f_{x_j}(t)\right] = \delta_j \frac{Cov_u(\lambda^d_{x_i}(t), \lambda^d_{x_j}(t))}{\sqrt{Var_u(\lambda^d_{x_i}(t)) \cdot Var_u(\lambda^f_{x_j}(t))}}, \tag{6}$$

where

- $Cov_u(\lambda^d_{x_i}(t), \lambda^d_{x_j}(t))$ is computed using the Gaussian mapping technique,
- $Var_u(\lambda^f_{x_j}(t)) = \delta_j^2 Var_u(\lambda^d_{x_j}(t)) + (1-\delta_j)^2 Var_u(\lambda'(t; x_j))$.

Gaussian Mapping | Variance

Annuity Contracts

Let

- n_i be the number of annuities sold to heads aged x_i, for $i = 1, \dots, m$,
- $N_i(t)$ be the value at time t of the annuity contract sold initially to heads aged x_i.

If the portfolio is composed by annuities that pay the annual installment R, then its actuarial value $AV_\Pi(t)$: is:

$$AV_\Pi(t) = \sum_{i=1}^{m} n_i N_i(t), \quad \text{with} \tag{7}$$

$$N_i(t) = R \sum_{u=1}^{\omega - t} D(t, t+u) S_i(t, t+u), \tag{8}$$

where D is the discount factor and S_i the survival probability.

Annuity Portfolio

- $\Pi(t)$: portfolio value at time t
- $RM_{\Pi}(t)$: Portfolio Risk Margin

$$\Pi(t) = AV_{\Pi}(t) + RM_{\Pi}(t). \tag{9}$$

Risk Margin

The portfolio risk margin $RM_{\Pi}(t)$ is defined as the discounted Value-at-Risk, at a certain confidence level $\alpha \in (0,1)$ - say $\alpha = 0.005$ - of the unexpected portfolio's future actuarial value increase at a given time horizon T:

$$\begin{aligned} RM_{\Pi}(t) &= D(t, t+T) \cdot VaR_{\alpha}\Big(AV_{\Pi}(t+T) - \mathbb{E}_t[AV_{\Pi}(t+T)]\Big), \qquad (10) \\ &= D(t, t+T) \cdot inf\{l \in \mathbb{R}^+ : P(AV_{\Pi}(t+T) - \mathbb{E}_t[AV_{\Pi}(t+T)] > l) < 1 - \alpha\}. \end{aligned}$$

Annuity Portfolio Expansion

We consider the case of an Insurer that has an annuity portfolio exposed to the domestic population

$$\Pi^0 = AV_{\Pi^0} + RM_{\Pi^0}$$

and can choose between a domestic or a foreign expansion:

- Acquiring a new domestic portfolio Π^0, ending up with $\Pi^1 = \Pi^0 + \Pi^0$,
- Acquiring a new foreign portfolio Π^F, ending up with $\Pi^2 = \Pi^0 + \Pi^F$.

Risk Margin Reduction:

$$\Delta RM_j = \frac{RM_{\Pi^0}}{AV_{\Pi^0}} - \frac{RM_{\Pi^j}}{AV_{\Pi^j}}, \quad j = 1, 2. \tag{11}$$

Similarity and Diversification Index

Let n_i^f be the number of annuities sold to cohort x_i in the foreign population, and let $n_i = n_i^d + n_i^f$ and m^f the number of generations in the foreign portfolio.

- **Similarity Index:**

$$SI = 1 - \frac{1}{m^f} \sum_{i=1}^{m^f} \left(1 - \frac{n_i^d + n_i^f \delta_i}{n_i} \right). \tag{12}$$

- **Diversification Index:**

$$DI = 1 - SI. \tag{13}$$

Property

1. If $\delta_i = 1$ for every $i \Rightarrow SI = 1$
2. If $\delta_i = 0$ for every i and $n_i^f \to \infty$ while n_i^d remains constant $\Rightarrow DI \to 1$

Empirical Application

Populations

Domestic Population

Foreign Population

Parameters Estimation

The estimation of parameters is performed using a 3-step procedure:

1. Estimate UK parameters $a_i, b_i, \sigma_i, \lambda_i(0)$:
   - We use 20 years of UK death rates data for each generation (1993-2012, source: HMD),
   - $\forall i, \lambda_i(0) = -ln(S_i)$, where S_i is the one-year observed survival probability of cohort x_i at time zero,
   - Minimize RMSE between the empirical and the model implied survival probabilities to obtain a_i, b_i, σ_i.
2. Estimate ITA parameters $a', b', \sigma', \delta_i, \lambda_i'(0)$:
   - We use 20 years of ITA death rates data for each generation (1993-2012, source: HMD),
   - Minimize RMSE between the empirical and the model implied survival probabilities, using all the parameters estimated at the previous step.
3. Estimate instantaneous correlations $\rho_{i,j}$ between UK generations:
   - We use 54 years of UK central mortality rates (period data) for each generation (1959-2012, source: HMD)
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2. Estimate ITA parameters $a', b', \sigma', \delta_i, \lambda'_i(0)$:
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Empirical Estimation: UK vs Italy

Table. Domestic population (UK) calibration results.

Age	a	b	σ	λ_0	RMSE
65	$2.7878 \cdot 10^{-5}$	0.0723	0.0075	0.0116	0.00035
66	$6.5423 \cdot 10^{-5}$	0.0652	0.0059	0.0124	0.00028
67	$1.8424 \cdot 10^{-5}$	0.0740	0.0080	0.0135	0.00035
68	$5.3144 \cdot 10^{-5}$	0.0685	0.0084	0.0160	0.00043
69	$1.2500 \cdot 10^{-4}$	0.0589	0.0091	0.0164	0.00039
70	$8.4734 \cdot 10^{-5}$	0.0646	0.0108	0.0189	0.00056
71	$7.1323 \cdot 10^{-5}$	0.0667	0.0106	0.0212	0.00038
72	$4.1759 \cdot 10^{-5}$	0.0688	0.0073	0.0239	0.00040
73	$2.2984 \cdot 10^{-5}$	0.0689	0.0066	0.0262	0.00063
74	$9.6036 \cdot 10^{-5}$	0.0663	0.0131	0.0282	0.00040
75	$3.3898 \cdot 10^{-5}$	0.0684	0.0077	0.0316	0.00049

Empirical Estimation: UK vs Italy

Table. Foreign population (ITA) calibration results.

Age	a'	b'	σ'	δ	RMSE	λ'_0
65				0.8071	0.00060	0.0075
66				0.8036	0.00073	0.0127
67				0.9348	0.00031	0.0190
68				0.8074	0.00045	0.0115
69				0.7893	0.00120	0.0163
70	$5.8458 \cdot 10^{-5}$	$4.2841 \cdot 10^{-11}$	$1.1464 \cdot 10^{-7}$	0.8119	0.00053	0.0141
71				0.7903	0.00099	0.0124
72				0.8006	0.00039	0.0092
73				0.8106	0.00064	0.0115
74				0.7622	0.00160	0.0209
75				0.8470	0.00053	0.0182

Empirical Estimation: UK vs Italy

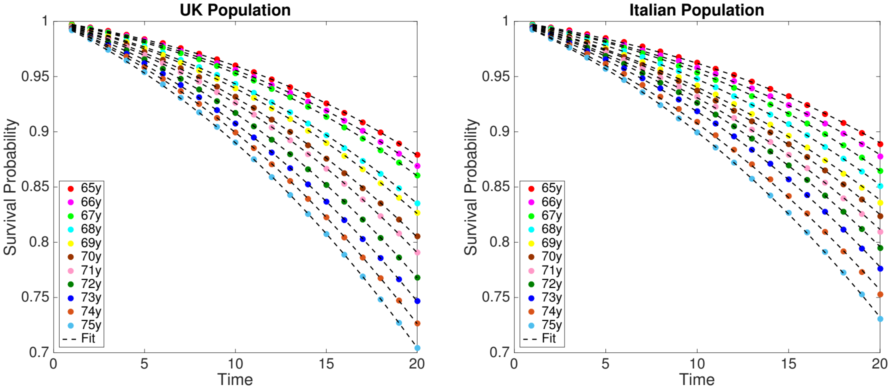

Figure. Fit of Survival probabilities.

Empirical Estimation: UK vs Italy

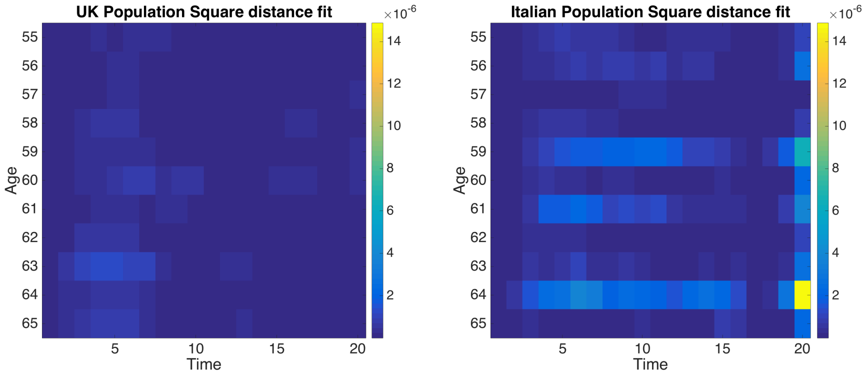

Figure. Calibration error.

Instantaneous Correlation Matrix UK

Table. Instantaneous correlation matrix UK population. Colored cells highlight the minimum of each column.

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
65	1										
66	0.9990	1									
67	0.9983	0.9992	1								
68	0.9983	0.9988	0.9989	1							
69	0.9973	0.9985	0.9988	0.9993	1						
70	0.9969	0.9979	0.9983	0.9989	0.9995	1					
71	0.9972	0.9977	0.9983	0.9987	0.9988	0.9986	1				
72	0.9964	0.9970	0.9977	0.9984	0.9987	0.9986	0.9994	1			
73	0.9962	0.9970	0.9976	0.9985	0.9988	0.9989	0.9992	0.9997	1		
74	0.9959	0.9967	0.9974	0.9983	0.9989	0.9991	0.9991	0.9995	0.9996	1	
75	0.9957	0.9960	0.9964	0.9974	0.9978	0.9981	0.9990	0.9996	0.9994	0.9995	1

Gaussian Mapping

Correlation between populations

Table. Correlation between populations. Rows are UK generations, columns are Italian generations. Colored cells highlight the minimum of each row.

	65	66	67	68	69	70	71	72	73	74	75
65	0.9821	0.9815	0.9803	0.9807	0.9803	0.9797	0.9798	0.9790	0.9789	0.9786	0.9785
66	0.9815	0.9830	0.9817	0.9817	0.9820	0.9812	0.9809	0.9800	0.9801	0.9799	0.9793
67	0.9803	0.9817	0.9819	0.9812	0.9817	0.9810	0.9809	0.9802	0.9802	0.9801	0.9795
68	0.9807	0.9817	0.9812	0.9827	0.9826	0.9820	0.9816	0.9813	0.9815	0.9813	0.9805
69	0.9803	0.9820	0.9817	0.9826	0.9839	0.9831	0.9823	0.9822	0.9824	0.9825	0.9814
70	0.9797	0.9812	0.9810	0.9820	0.9831	0.9834	0.9819	0.9819	0.9822	0.9823	0.9815
71	0.9798	0.9809	0.9809	0.9816	0.9823	0.9819	0.9831	0.9825	0.9824	0.9823	0.9823
72	0.9790	0.9800	0.9802	0.9813	0.9822	0.9819	0.9825	0.9830	0.9828	0.9826	0.9827
73	0.9789	0.9801	0.9802	0.9815	0.9824	0.9822	0.9824	0.9828	0.9832	0.9829	0.9826
74	0.9786	0.9799	0.9801	0.9813	0.9825	0.9823	0.9823	0.9826	0.9829	0.9832	0.9827
75	0.9785	0.9793	0.9795	0.9805	0.9814	0.9815	0.9823	0.9827	0.9826	0.9827	0.9833

Covariance matrix between populations

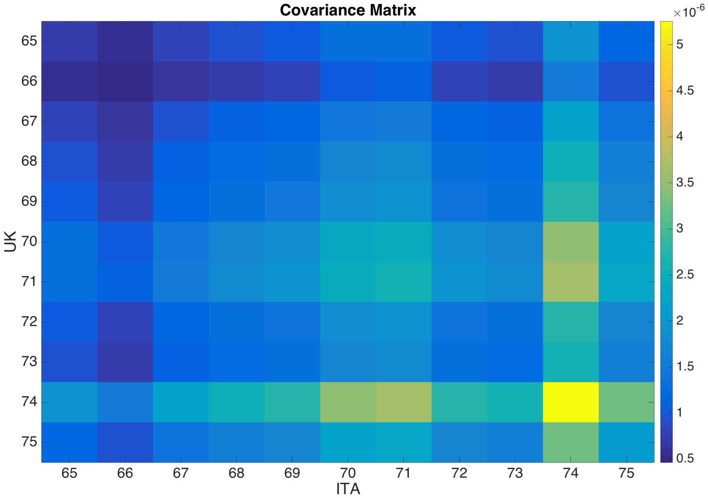

Figure. Covariance matrix between Italian and UK generations.

Effects of Geographical Diversification

Table. Effects of geographical diversification ($r = 2\%$)

Portfolio	*AV*	*RM*	Π	$\%RM$	*DI*
Π^0	$1.5288 \cdot 10^4$	$1.3018 \cdot 10^3$	$1.6590 \cdot 10^4$	8.52%	-
Π^F	$1.5964 \cdot 10^4$	$1.1584 \cdot 10^3$	$1.7123 \cdot 10^4$	7.26%	-
Π^1	$3.0576 \cdot 10^4$	$2.6036 \cdot 10^3$	$3.3179 \cdot 10^4$	8.52%	0
Π^2	$3.1252 \cdot 10^4$	$2.4602 \cdot 10^3$	$3.3712 \cdot 10^4$	7.87%	0.0925
Π^3	$4.7217 \cdot 10^4$	$3.6186 \cdot 10^3$	$5.0835 \cdot 10^4$	7.66%	0.1233
Π^1_{opt}	$3.2979 \cdot 10^4$	$2.1947 \cdot 10^3$	$3.5173 \cdot 10^4$	6.65%	0
Π^2_{opt}	$3.3447 \cdot 10^4$	$2.0621 \cdot 10^3$	$3.5509 \cdot 10^4$	6.17%	0.1801

- Π^0 is the initial portfolio:

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
UK	100	100	100	100	100	100	100	100	100	100	100
ITA	0	0	0	0	0	0	0	0	0	0	0

Effects of Geographical Diversification

Table. Effects of geographical diversification ($r = 2\%$)

Portfolio	AV	RM	Π	$\%RM$	DI
Π^0	$1.5288 \cdot 10^4$	$1.3018 \cdot 10^3$	$1.6590 \cdot 10^4$	8.52%	-
Π^F	$1.5964 \cdot 10^4$	$1.1584 \cdot 10^3$	$1.7123 \cdot 10^4$	7.26%	-
Π^1	$3.0576 \cdot 10^4$	$2.6036 \cdot 10^3$	$3.3179 \cdot 10^4$	8.52%	0
Π^2	$3.1252 \cdot 10^4$	$2.4602 \cdot 10^3$	$3.3712 \cdot 10^4$	7.87%	0.0925
Π^3	$4.7217 \cdot 10^4$	$3.6186 \cdot 10^3$	$5.0835 \cdot 10^4$	7.66%	0.1233
Π^1_{opt}	$3.2979 \cdot 10^4$	$2.1947 \cdot 10^3$	$3.5173 \cdot 10^4$	6.65%	0
Π^2_{opt}	$3.3447 \cdot 10^4$	$2.0621 \cdot 10^3$	$3.5509 \cdot 10^4$	6.17%	0.1801

- Π^F is the foreign portfolio:

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
UK	0	0	0	0	0	0	0	0	0	0	0
ITA	100	100	100	100	100	100	100	100	100	100	100

Effects of Geographical Diversification

Table. Effects of geographical diversification ($r = 2\%$)

Portfolio	AV	RM	Π	$\%RM$	DI
Π^0	$1.5288 \cdot 10^4$	$1.3018 \cdot 10^3$	$1.6590 \cdot 10^4$	8.52%	-
Π^F	$1.5964 \cdot 10^4$	$1.1584 \cdot 10^3$	$1.7123 \cdot 10^4$	7.26%	-
Π^1	$3.0576 \cdot 10^4$	$2.6036 \cdot 10^3$	$3.3179 \cdot 10^4$	8.52%	0
Π^2	$3.1252 \cdot 10^4$	$2.4602 \cdot 10^3$	$3.3712 \cdot 10^4$	7.87%	0.0925
Π^3	$4.7217 \cdot 10^4$	$3.6186 \cdot 10^3$	$5.0835 \cdot 10^4$	7.66%	0.1233
Π^1_{opt}	$3.2979 \cdot 10^4$	$2.1947 \cdot 10^3$	$3.5173 \cdot 10^4$	6.65%	0
Π^2_{opt}	$3.3447 \cdot 10^4$	$2.0621 \cdot 10^3$	$3.5509 \cdot 10^4$	6.17%	0.1801

- $\Pi^1 = \Pi^0 + \Pi^0$ is the portfolio after domestic expansion:

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
UK	200	200	200	200	200	200	200	200	200	200	200
ITA	0	0	0	0	0	0	0	0	0	0	0

Effects of Geographical Diversification

Table. Effects of geographical diversification ($r = 2\%$)

Portfolio	AV	RM	Π	$\%RM$	DI
Π^0	$1.5288 \cdot 10^4$	$1.3018 \cdot 10^3$	$1.6590 \cdot 10^4$	8.52%	-
Π^F	$1.5964 \cdot 10^4$	$1.1584 \cdot 10^3$	$1.7123 \cdot 10^4$	7.26%	-
Π^1	$3.0576 \cdot 10^4$	$2.6036 \cdot 10^3$	$3.3179 \cdot 10^4$	8.52%	0
Π^2	$3.1252 \cdot 10^4$	$2.4602 \cdot 10^3$	$3.3712 \cdot 10^4$	7.87%	0.0925
Π^3	$4.7217 \cdot 10^4$	$3.6186 \cdot 10^3$	$5.0835 \cdot 10^4$	7.66%	0.1233
Π^1_{opt}	$3.2979 \cdot 10^4$	$2.1947 \cdot 10^3$	$3.5173 \cdot 10^4$	6.65%	0
Π^2_{opt}	$3.3447 \cdot 10^4$	$2.0621 \cdot 10^3$	$3.5509 \cdot 10^4$	6.17%	0.1801

- $\Pi^2 = \Pi^0 + \Pi^F$ is the portfolio after foreign expansion:

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
UK	100	100	100	100	100	100	100	100	100	100	100
ITA	100	100	100	100	100	100	100	100	100	100	100

Effects of Geographical Diversification

Table. Effects of geographical diversification ($r = 2\%$)

Portfolio	*AV*	*RM*	Π	%*RM*	*DI*
Π^0	$1.5288 \cdot 10^4$	$1.3018 \cdot 10^3$	$1.6590 \cdot 10^4$	8.52%	-
Π^F	$1.5964 \cdot 10^4$	$1.1584 \cdot 10^3$	$1.7123 \cdot 10^4$	7.26%	-
Π^1	$3.0576 \cdot 10^4$	$2.6036 \cdot 10^3$	$3.3179 \cdot 10^4$	8.52%	0
Π^2	$3.1252 \cdot 10^4$	$2.4602 \cdot 10^3$	$3.3712 \cdot 10^4$	7.87%	0.0925
Π^3	$4.7217 \cdot 10^4$	$3.6186 \cdot 10^3$	$5.0835 \cdot 10^4$	7.66%	0.1233
Π^1_{opt}	$3.2979 \cdot 10^4$	$2.1947 \cdot 10^3$	$3.5173 \cdot 10^4$	6.65%	0
Π^2_{opt}	$3.3447 \cdot 10^4$	$2.0621 \cdot 10^3$	$3.5509 \cdot 10^4$	6.17%	0.1801

- $\Pi^3 = \Pi^0 + 2\Pi^F$ is the portfolio after a more aggressive foreign expansion:

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
UK	100	100	100	100	100	100	100	100	100	100	100
ITA	200	200	200	200	200	200	200	200	200	200	200

Effects of Geographical Diversification

Table. Effects of geographical diversification ($r = 2\%$)

Portfolio	AV	RM	Π	$\%RM$	DI
Π^0	$1.5288 \cdot 10^4$	$1.3018 \cdot 10^3$	$1.6590 \cdot 10^4$	8.52%	-
Π^F	$1.5964 \cdot 10^4$	$1.1584 \cdot 10^3$	$1.7123 \cdot 10^4$	7.26%	-
Π^1	$3.0576 \cdot 10^4$	$2.6036 \cdot 10^3$	$3.3179 \cdot 10^4$	8.52%	0
Π^2	$3.1252 \cdot 10^4$	$2.4602 \cdot 10^3$	$3.3712 \cdot 10^4$	7.87%	0.0925
Π^3	$4.7217 \cdot 10^4$	$3.6186 \cdot 10^3$	$5.0835 \cdot 10^4$	7.66%	0.1233
Π^1_{opt}	$3.29791 \cdot 10^4$	$2.1947 \cdot 10^3$	$3.5173 \cdot 10^4$	6.65%	0
Π^2_{opt}	$3.3447 \cdot 10^4$	$2.0621 \cdot 10^3$	$3.5509 \cdot 10^4$	6.17%	0.1801

- Π^1_{opt} is the portfolio after domestic expansion with optimal composition:

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
UK	100	1200	100	100	100	100	100	100	100	100	100
ITA	0	0	0	0	0	0	0	0	0	0	0

Effects of Geographical Diversification

Table. Effects of geographical diversification ($r = 2\%$)

Portfolio	AV	RM	Π	$\%RM$	DI
Π^0	$1.5288 \cdot 10^4$	$1.3018 \cdot 10^3$	$1.6590 \cdot 10^4$	8.52%	-
Π^F	$1.5964 \cdot 10^4$	$1.1584 \cdot 10^3$	$1.7123 \cdot 10^4$	7.26%	-
Π^1	$3.0576 \cdot 10^4$	$2.6036 \cdot 10^3$	$3.3179 \cdot 10^4$	8.52%	0
Π^2	$3.1252 \cdot 10^4$	$2.4602 \cdot 10^3$	$3.3712 \cdot 10^4$	7.87%	0.0925
Π^3	$4.7217 \cdot 10^4$	$3.6186 \cdot 10^3$	$5.0835 \cdot 10^4$	7.66%	0.1233
Π^1_{opt}	$3.29791 \cdot 10^4$	$2.1947 \cdot 10^3$	$3.5173 \cdot 10^4$	6.65%	0
Π^2_{opt}	$3.3447 \cdot 10^4$	$2.0621 \cdot 10^3$	$3.5509 \cdot 10^4$	6.17%	0.1801

- Π^2_{opt} is the portfolio after foreign expansion with optimal composition:

	65	**66**	**67**	**68**	**69**	**70**	**71**	**72**	**73**	**74**	**75**
UK	100	100	100	100	100	100	100	100	100	100	100
ITA	0	1100	0	0	0	0	0	0	0	0	0

Effects of Geographical Diversification

Table. Effects of geographical diversification ($r = 0\%$)

Portfolio	AV	RM	Π	$\%RM$	DI
Π^0	$1.9097 \cdot 10^4$	$2.1318 \cdot 10^3$	$2.1228 \cdot 10^4$	11.16%	-
Π^F	$2.0093 \cdot 10^4$	$1.9060 \cdot 10^3$	$2.1999 \cdot 10^4$	9.49%	-
Π^1	$3.8193 \cdot 10^4$	$4.2636 \cdot 10^3$	$4.2457 \cdot 10^4$	11.16%	0
Π^2	$3.9189 \cdot 10^4$	$4.0378 \cdot 10^3$	$4.3227 \cdot 10^4$	10.30%	0.0925
Π^3	$5.9282 \cdot 10^4$	$5.9437 \cdot 10^3$	$6.5226 \cdot 10^4$	10.03%	0.1233
Π^1_{opt}	$4.1675 \cdot 10^4$	$3.6480 \cdot 10^3$	$4.5323 \cdot 10^4$	8.75%	0
Π^2_{opt}	$4.2400 \cdot 10^4$	$3.4234 \cdot 10^3$	$4.5824 \cdot 10^4$	8.07%	0.1801

Conclusions

Conclusions

The empirical application shows that:

- Our proposed model:
  - Fits well the empirical data,
  - Has endogenous correlations within and across populations but a parsimonious number of parameters as a whole,
  - Allows to compute similarity and diversification indices of insurance companies' liability portfolios,
- Geographical diversification reduces risk margins,
- The magnitude of the reduction depends on the similarities of the two populations,
- Low interest rates amplify the effect of geographical diversification.

Thanks!

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Gompertz Mortality

$$d\lambda_x^d(t) = (a + b\lambda_x^d(t))dt + \sigma\sqrt{\lambda_x^d(t)}dW(t) \tag{14}$$

- If $a = \sigma = 0$, then the mortality intensity is deterministic and we have:

$$d\lambda_x^d(t) = b\lambda_x^d(t)dt, \tag{15}$$

- that after simple integration becomes:

$$\lambda_x^d(t) = \lambda_x^d(0)e^{bt} \tag{16}$$

- which is the usual **Gompertz model**.

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Delta

$$\lambda_x^f = \delta \underbrace{\lambda_x^d}_{\textbf{Common Factor}} + (1-\delta) \underbrace{\lambda_x'}_{\textbf{Idiosyncratic Factor}}, \tag{17}$$

The parameter δ measures the dependence between the two populations:

1. $\delta = 1 \Rightarrow$ The two population are the same $\Rightarrow$ perfect dependence
2. $0 < \delta < 1 \Rightarrow$ The two population are different $\Rightarrow$ partial dependence
3. $\delta = 0 \Rightarrow$ The two population are different $\Rightarrow$ perfect independence

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Idiosyncratic component specification

Specification 2: A different specification for λ'_{x_i} is:

$$a(x_i) = a' x_i,$$
$$b(x_i) = b',$$
$$\sigma(x_i) = \sigma' e^{\gamma' x_i},$$

with $a', b', \sigma', \gamma' > 0$.

- For each x_i, λ'_{x_i} is different but has the same functional form and the same set of parameters. This allows the model to be parsimonious.
- Since $a' > 0$, the drift of the mortality intensity is increasing with age.
- $\gamma' > 0$ captures the empirical evidence that the volatility of mortality tends to increase with age (*see also Fung et al. (2014)*).

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Variance

$$Var_u(\lambda^d_{x_i}(t)) = \frac{a_i\sigma_i^2}{2b_i^2}\left(e^{b_i(t-u)} - 1\right)^2 + \frac{\sigma_i^2}{b_i} e^{b_i(t-u)}\left(e^{b_i(t-u)} - 1\right)\lambda^d_{x_i}(u) \tag{18}$$

$$\begin{aligned} Var_u(\lambda'(t;x_i)) = {} & \frac{a(x_i;a')\sigma(x_i;\sigma',\gamma')^2}{2b(x_i;b')^2}\left(e^{b(x_i;b')(t-u)} - 1\right)^2 + \\ & + \frac{\sigma(x_i;\sigma',\gamma')^2}{b(x_i;b')} e^{b(x_i;b')(t-u)}\left(e^{b(x_i;b')(t-u)} - 1\right)\lambda'(u;x_i) \end{aligned} \tag{19}$$

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Gaussian Mapping [3]

For each generation x_i, we map the CIR dynamic

$$d\lambda_{x_i}^d = (a_i + b_i \lambda_i^d)dt + \sigma_i \sqrt{\lambda_i^d} dW_i$$

into a Vasicek dynamics which is as "close" as possible, i.e

$$d\lambda_i^V = (a_i + b_i \lambda_i^V)dt + \sigma_i^V dW_i, \quad \lambda_i^V(0) = \lambda_i^d(0),$$

where σ_i^V is such that

$$S_i^d(t, T) = S_i^V(t, T; \sigma_i^V).$$

$$\Rightarrow Corr_0(\lambda_i^d, \lambda_j^d) \approx Corr_0(\lambda_i^V, \lambda_j^V)$$

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[3] For more details see Brigo and Mercurio (2001)

Gaussian Mapping [4]

For each generation x_i, we map the CIR dynamic

$$d\lambda_{x_i}^d = (a_i + b_i\lambda_i^d)dt + \sigma_i\sqrt{\lambda_i^d}dW_i$$

into a Vasicek dynamics which is as "close" as possible, i.e

$$d\lambda_i^V = (a_i + b_i\lambda_i^V)dt + \sigma_i^V dW_i, \quad \lambda_i^V(0) = \lambda_i^d(0),$$

where σ_i^V is such that

$$S_i^d(t, T) = S_i^V(t, T; \sigma_i^V).$$

$$\Rightarrow Corr_0(\lambda_i^d, \lambda_j^d) \approx Corr_0(\lambda_i^V, \lambda_j^V)$$

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[4] For more details see Brigo and Mercurio (2001)