Problem Set 2

Due Wednesday 21st, April.

1. Consider the following Robinson Crusoe economy. Robinson the consumer is endowed with zero units of coconuts, x, and 24 hours of time, h, so that $\mathbf{e} = (0, 24)$. His preferences are defined over $\mathbb{R}^2_+$ and represented by $u(x, h) = x^{\frac{1}{3}} h^{\frac{2}{3}}$. Robinson the producer uses the consumer's labor services, l, to produce coconuts, y, according to the production function $y = \sqrt{l}$. The producer sells the coconuts to the consumer. All profits from the production and sale of coconuts are distributed to the consumer. Find the Walrasian equilibrium prices and allocation of this economy. Argue that this equilibrium is Pareto Efficient.

2. Consider the following economy. There are two firms, firm 1 and firm 2. Firm 1 produces commodity 1 out of labor, l, according to the production function $y_1 = \sqrt{l}$. Firm 2 produces commodity 2 out of l according to the production function $y_2 = l$. There are two agents, A and B, with identical utility function $u(x_1, x_2, h) = x_1 x_2$, where, x_k, $k = 1, 2$, denotes commodity k, and h denotes the leisure time. Each consumer is endowed with 6 units of time. There is no initial endowment of any of the two commodities. Finally, both consumers own half of each firm. Compute the Walrasian equilibrium prices and allocation.

3. Consider a two-consumer, two-good exchange economy with externalities. Utility functions and endowments are:

$$\begin{aligned} u^A\left(x_1^A, x_2^A\right) &= x_1^A + 2\ln\left(x_2^A\right) & \text{and} \quad \mathbf{e}^A = (12, 15), \\ u^B\left(x_1^B, x_2^B, x_2^A\right) &= x_1^B + 2\ln\left(x_2^B\right) - \ln x_2^A & \text{and} \quad \mathbf{e}^B = (3, 5). \end{aligned}$$

   (a) Prove that if $x = \left(\left(x_1^A, x_2^A\right), \left(x_1^B, x_1^B\right)\right)$ is a Pareto efficient allocation with $x_1^A > 0$ and $x_1^B > 0$, then it must be that: $x_2^A = \frac{20}{3}$ and $x_2^2 = \frac{40}{3}$.

   (b) Find the Walrasian equilibrium prices and allocation. Is the equilibrium allocation Pareto efficient? Explain.

   (c) Consider the efficient allocation $x^* = \left(\left(13, \frac{20}{3}\right), \left(2, \frac{40}{3}\right)\right)$. Suppose that the government can tax consumer A with a (Pigouvian) per unit tax on consumption of good 2 . Furthermore, the government can redistribute revenue on a lump-sum basis in the amounts T_i, $i \in \{A, B\}$. Let t denote the per-unit tax (i.e., consumer 2 pays t for each unit of commodity 2 that he buys). Find T_A^*, T_B^* and t^* such that the Walrasian equilibrium allocation is x^*. Specify the Walrasian equilibrium prices. Is the government's budget constraint satisfied with equality? Explain.