Problem Set 6 – Signaling Games
Microeconomics II (Allievi Programme)

1. Solve Mas-Colell, Whinston & Green 13.C.5. (Hint: the answer to part b is *no*; to prove non-existence of a separating PBE you should use the fact that $c_H > c_L$.)

   The idea for the solution is as follows: The high type must decide whether or not to spend a constant amount A in advertising. Since $c_H > c_L$, if it were optimal for H to do it, so would be for L. Hence, spending A cannot possibly be a signal that reveals the type; since either both H and L will do it, or nobody will do it... but it never will be done by H alone, which is that is needed for a PBE to arise. (Remember the idea that marginal cost for H types should be smaller than for L for signalling to work, which is violated in this exercise because $c_H > c_L$.)

2. Consider the signaling model with two types of workers presented in Mas-Colell *et al*, section 13.C. Assume that $\theta_L = 1$ and $\theta_H = 4$. Workers can acquire education before entering the labour market; education does not change workers' productivity. Let the cost function for education be: $c(e, \theta) = e^2/(3\theta)$. The utility of a worker of type θ is then given by: $u(w, e \mid \theta) = w - e^2/(3\theta)$.
   i) Find the lowest and highest possible levels of education chosen by the types-H in a separating PBE.
   ii) Let $\lambda \in (0, 1)$ denote the fraction of types-H in the economy. For which values of λ might the types-H be better-off when they can use schooling to signal their intrinsic ability?

   i) These are the results from solving the following equations:

   $$\text{lowest: } 1 = 4 - \frac{\widetilde{e}^2}{3} \quad \Rightarrow \quad \widetilde{e} = 3.$$

   $$\text{highest: } 1 = 4 - \frac{\widehat{e}^2}{12} \quad \Rightarrow \quad \widehat{e} = 6.$$

   ii) When there is no schooling, everybody is offered the average wage (assuming $r(\theta) = 0$ for all θ); that is: $w_{ns} = 4\lambda + (1 - \lambda)1 = 3\lambda + 1$. So, a type H may be better off with the possibility of use of schooling as a signal only if:

   $$3\lambda + 1 < 4 - \frac{\widetilde{e}^2}{12}.$$

3. Consider again the signaling model with two types of workers presented in Mas-Colell *et al*, section 13.C. Let $\widetilde{e}$ denote the lowest possible level of education chosen by the types-H in a separating PBE. Show that in any separating PBE it must necessarily be the case that $\mu(e) < 1$ for all $e < \widetilde{e}$. (Hint: solve graphically.)

   For a solution, you should prove by contradiction. Assume that $\mu(e_0) = 1$, for some $e_0 < \widetilde{e}$. Then, the zero-profit condition for firms, given those beliefs, implies that $w(e_0) = \theta_H$. But since $\widetilde{e}$ solves:

   $$\theta_L = \theta_H - c(\widetilde{e} \mid \theta_L),$$

   and $c(\widetilde{e} \mid \theta_L) > c(e_0 \mid \theta_L)$, then:

   $$\theta_L < \theta_H - c(e_0 \mid \theta_L),$$

   implying that types-L would also optimally choose to get e_0 education if $\mu(e_0) = 1$. As a result, $\mu(e_0) = 1$ cannot be true as it violates Bayes rule, given the optimal behaviour of agents. (A GRAPHIC PROOF IS MUCH SIMPLER AND ILLUSTRATIVE.)