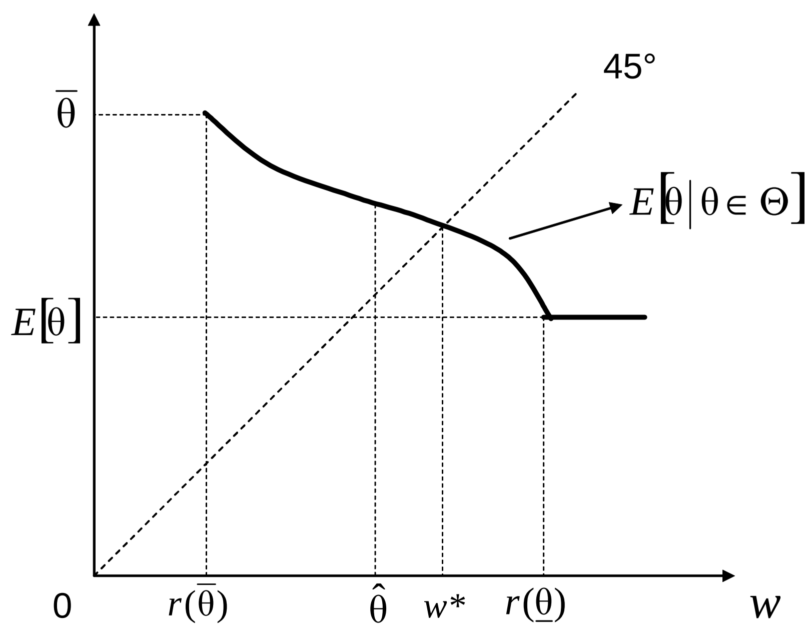
Problem Set 5 – Solutions

1. **M-C, W & G 13.B.3**

a) Suppose firms offer a wage $\widehat{w}$ and there exists $\widehat{\theta}$ such that $r(\widehat{\theta}) = \widehat{w}$. Then, since $r'(\cdot) < 0$, $r(\theta) < \widehat{w}$ for all $\theta > \widehat{\theta}$, implying that the set of workers who choose to work is $\Theta = \left\{\theta : \theta \in \left(\widehat{\theta}, \overline{\theta}\right)\right\}$.

b) First of all, notice that only wages $w \in \left[\underline{\theta}, \overline{\theta}\right]$ may prevail in a competitive equilibrium, as they must equal the *conditional* average productivity $E(\theta \mid \theta \in \Theta)$, which necessarily lies within the interval $\left[\underline{\theta}, \overline{\theta}\right]$. Secondly, observe that, since $r(\overline{\theta}) > \overline{\theta}$ and $r'(\cdot) < 0$, for any $w \in \left[\underline{\theta}, \overline{\theta}\right]$, it follows that $r(\theta) > \overline{\theta}$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$. As a result, in a competitive equilibrium $\Theta^* = \emptyset$, which is indeed Pareto-efficient since $r(\theta) > \theta$ for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$.

c) A graphic representation of a competitive equilibrium with positive employment (one possible representation!) looks as follows:



From the graph we can immediately observe that the equilibrium wage $w^* > \widehat{\theta}$, implying that there exist some $\theta < \widehat{\theta}$ who belong to Θ^*. As a result, there exist some $\theta \in \Theta^*$, such that $r(\theta) > \theta$, who should not be working for firms in a Pareto-efficient allocation of workers.

Remark 1. Notice that this is a general result *regardless* of the exact graphic representation used for $E\left[\theta \mid \theta \in \Theta\right]$. (Try by yourselves different graphic representations to see that.) This is the case, because when $w = \widehat{\theta}$, then $\Theta = \left[\widehat{\theta}, \overline{\theta}\right]$; therefore $E\left[\theta \mid \theta \geq \widehat{\theta}\right] > w$, which yields strictly positive expected profits to firms and thus cannot represent a competitive equilibrium.

2. When deciding whether or not to sell their car, *informed* car owners compare the market price to their *own* reservation price, θ. Conversely, *uninformed* car owners compare the market price to the *average* reservation price, $E[\theta]$. Assume that car owners sell their cars in case they are indifferent between selling or not. The supply for cars is then given by:

$$S(P) = \begin{cases} (1-\delta)\,F(P) + \delta, & \text{if} \quad P \geq E[\theta]\,; \\ (1-\delta)\,F(P), & \text{if} \quad P < E[\theta]\,. \end{cases} \tag{1}$$

On the other hand, when deciding whether or not to buy a car, the potential buyers compare their *expected* utility (conditional on all their information they possess) to the market price. As a result, the demand for cars reads as follows (N.B. that the mass of non-owners is $N > 1$):

$$D(P) = \begin{cases} 0, & \text{if} \quad P > E[\upsilon(\theta) \mid P]\,; \\ [0, N]\,, & \text{if} \quad P = E[\upsilon(\theta) \mid P]\,; \\ N, & \text{if} \quad P < E[\upsilon(\theta) \mid P]\,. \end{cases} \tag{2}$$

Where,

$$E[\upsilon(\theta) \mid P] = \begin{cases} (1-\delta)\; E[\upsilon(\theta) \mid \theta \leq P] + \delta\, E[\upsilon(\theta)]\,, & \text{if} \quad P \geq E[\theta]\,; \\ E[\upsilon(\theta) \mid \theta \leq P] & \text{if} \quad P < E[\theta]\,. \end{cases} \tag{3}$$

Remark 2. A *very* subtle, but important implicit assumption when writing (1) –which in turn affects (2) and (3)– is the fact that the proportion of uninformed sellers δ is identical for all values of $\theta \in [\underline{\theta}, \overline{\theta}]$. Otherwise, we should take into account the correlation between being an informed seller and the quality of the car the seller owns.

3. In class.