Problem Set 2

1. Take the model in Galor & Zeira (1993) exactly as presented in class (i.e. keep all parametric assumptions and restrictions exactly as presented). Assume that the parameter $\beta = \infty$ (see Eq.(6) in the article). Describe the dynamics of the model in this particular case. Interpret the results (in terms of economic intuition).

2. Take the model in Galor & Zeira (1993) presented in class, but assume $(1-\alpha)(1+r) > 1$. Plot the dynamics of the model in this particular case. Explain intuitively.

3. **Unobservable Effort and Credit Rationing [based on Piketty (REStud, 1997)]**: Assume there exists a continuum of individuals i with utility $U_i = y_i - e_i$, where y_i denotes the income generated by i and $e_i = \{0, 1\}$ their effort. Individual i is born with a wealth endowment $w_i \geq 0$. There exists two investment opportunities available to agents. First, they can deposit their wealth (or part of it) in banks, receiving $(1+r)\,D_i$ in return, where $r \geq 0$ and $0 \leq D_i \leq w_i$ is the amount deposited in the banks by i. Alternatively, individuals can invest an amount of capital $k_i > 0$ into a risky project, and produce:

$$f(k_i) = \begin{cases} k_i^{\alpha} & \text{with prob. } p \\ 0 & \text{with prob. } 1-p \end{cases} \text{ if } e_i = 1,$$

$$f(k_i) = 0 \text{ if } e_i = 0;$$

where $\alpha \in (0,1)$ and $p \in (0,1)$.

For simplicity, let now $r = 0$. In addition, assume that:

$$p^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} (1-\alpha) > 1. \tag{1}$$

i) Show that condition (1) implies that in the first-best *all* individuals should invest $k_i = k^* \equiv (\alpha p)^{\frac{1}{1-\alpha}}$ in the risky projects and exert high effort (i.e., $e_i = 1$)

Assume, now, that e_i is private information, while w_i is publicly observable. In addition, suppose $w_i < k^*$ for all individuals i. Also, assume individuals are protected by limited liability, which implies that $y_i \geq 0$. Finally, assume the banking industry is perfectly competitive, so banks must make zero (expected) profits in equilibrium. A bank will then lend an amount $L_i = (k^* - w_i)$ to individual i, offering a contract that stipulates (contingent) repayments (d_f, d_s) in case of failure (f) and success (s), respectively.

ii) Show that $d_f = 0$. How about d_s?

iii) Show that only sufficiently rich people receive credit from banks. Find the wealth threshold below which individuals are credit constrained.

4. Take the Banerjee & Newman (1993) model presented in class, but with the moral hazard problem tweaked in the following way: after borrowing an amount $(I - a_i)$, the borrower i may attempt to run away with the money and never pay back his loan. In case he does so, he will be caught with probability $\beta \in (0,1)$, in which case all his income/earnings will be taken away from him.

   i) Write down the Incentive Compatibility constraint of this problem. Who is able to borrow?

   ii) Suppose that actually β is an increasing function of the size of the loan; i.e.: $\beta = \Phi(I - a_i)$, with $\Phi(\cdot) : [0, I] \to [0, 1]$ and $\Phi'(\cdot) > 0$. Write down the now IC constraint and interpret it. Next, solve fully by letting $\Phi(I - a_i) = (I - a_i)/I$. Compare your results with those in point *i*) [**Warning**: for this particular case, it's important to bear in mind the only two *feasible* equilibrium wages].

5. Take the Banerjee & Newman (1993) model presented in class with the following parametric values: $r = 1, I = 1, \underline{\omega} = 1, q = 10, s = \frac{1}{3}, F = 2, \Pi = 0$. Assume that in the *Genesis* of this economy (i.e., in period $t = 0$) the initial wealth of dynasties is distributed uniformly along the interval $\left[0, \frac{3}{2}\right]$. In other words, $a_{i,0} \sim \left[0, \frac{3}{2}\right]$, so the density function in $t = 0$ is given by: $g_0(a) = \frac{2}{3}$, for all $0 \leq a \leq \frac{3}{2}$.

i) Find the equilibrium in $t = 0$. Depict the dynamic behaviour of this economy. Find the long-run wealth distribution in the economy.

ii) Suppose now that, at some $t = t' > 0$, there is an improvement in the technology of chasing fraudsters (e.g., faster police cars appear at zero cost). So, from t' onwards, $\Pi' = \frac{1}{4}$. Describe what happens in the economy after policemen start driving faster cars. Find the long-run wealth distribution of the fast-police-car economy? Are faster police cars Pareto improving?