Problem Set 1

1. **Solow Model with Subsistence Consumption:** Take a closed economy *almost* a la Solow. For simplicity, assume that all individuals are born with an identical endowment of one unit of labour time, and that all firms are own entirely by individuals who hold exactly the same number of shares of each firm: in other words, there is no income inequality in the economy, and all individuals receive an income equal to the GDP per capita of the economy. Also, for simplicity, assume the population is constant over time, and normalise its size to one; i.e. $L(t) = 1$.

   The only (major) difference with the standard textbook Solow model lies in that the saving rate is non-constant. In particular, all individuals aim to achieve (at least) a threshold level of consumption $\overline{c} > 0$. Having met that level of consumption, individuals start to save a fraction $s \in (0, 1)$ of each additional unit of income. More precisely, the consumption function reads as follows:

$$c = \begin{cases} y & \text{if } \ y \leq \overline{c}, \\ (1-s)\,(y-\overline{c}) + \overline{c} & \text{if } \ y > \overline{c}. \end{cases}$$

   Suppose the production function (in per capita terms) is given by: $y = Ak^{0.5}$.

   (a) Argue that if $\overline{c}$ is sufficiently large then (quite ironically!) all individuals are eventually doomed to starvation.

   (b) Let $A = 1, \delta = 0.05$ and $s = 0.1$. Are there values of $\overline{c} > 0$ for which economies might sustain positive consumption in the long run? Under which initial condition will economies reach that/those long-run equilibrium/equilibria.

2. **Solow Model with CES Production Function:** Take the standard Solow model, but assume that the aggregate production function is given by

$$Y = F(K, L) = A\left(K^{\psi} + L^{\psi}\right)^{\frac{1}{\psi}},$$

   where $1/(1-\psi)$ equals the elasticity of substitution between K and L and $\psi < 1$. Assume A is constant over time (ie. there is no technical change). Notice that when $\psi \to 0$, then $F(K, L)$ approaches a Cobb-Douglas production function.

   (a) Is it possible for income per capita, $y \equiv Y/L$, to actually grow continuously in the long run even in the absence of technical change?

   (b) The economy may be doomed to starvation in the long run. Discuss.

3. Take the model presented in *Section V* of Murphy, Shleifer and Vishny (1989).

   (a) Considering the parameter θ in the utility function, equation (11) in the text, discuss the following assertion: If, for a given set of parameters, an economy with $\theta = \theta_0 < 1$ is able to attain a positive-growth equilibrium, then an economy with the *same* parameters *except* for $\theta = \theta_1 > \theta_0$ must also be able to attain positive-growth equilibrium. True or false. Can you give some economic intuition for your answer?

(b) Suppose that actually $\theta = 1$ and, to simplify, also assume that $\beta = 1$ and $L = 1$. (Naturally, then $F < 1$.) Show that if $(\alpha - 1)/\alpha < F < (\alpha - 1)$, there exists *three* Nash equilibria in the model. In particular, show that we have the two equilibria discussed in the lecture (i.e., one in which no monopolist invests in improving the technology in the first period, and one in which all monopolists invest in that). But, in addition to those two equilibria, there exists another Nash equilibrium in which a fraction $0 < n^* < 1$ chooses to invest in improving the technology, while a fraction $(1 - n^*)$ decides not to invest in that.

4. Take the model presented in *Section IV* of Murphy, Shleifer and Vishny (1989), but tweaked in the following two ways: *i*) there is no factory wage premium (i.e., $v = 0$); *ii*) the set up (labour) cost of an urban factory is decreasing in the number of urban producers; in particular $F(n) = F/n$, where $0 \leq n \leq 1$ is the mass of industrial entrepreneurs.

   Note: We could think for example that there is fixed level of social infrastructure (like: roads, electricity, communication, etc.) that must be first set up in order for industrial production to be feasible. This social infrastructure has the features of a 'public good', hence its cost will be shared by all industrial producers.

   Assume that $F < L(\alpha - 1)/\alpha$, and find all the Nash equilibrium/equilibria of the model.