Problem Set 1 – Microeconomics II

Solutions will be posted on line on Tuesday 6th, April.

1. Solve Mas-Colell, Whinston & Green 14.B.4 (a), (b), and (c).

2. Consider a risk-neutral principal analysing how to design the optimal contract in order to induce a worker to exert high effort in his task. For simplicity, suppose there are two efforts levels: $e_L < e_H$. Output, x, can take the following 3 values: 0, 2, and 4. Output realisation probabilities are as follows: $p(0 \mid e_L) = 0.5$, $p(2 \mid e_L) = p(4 \mid e_L) = 0.25$; and $p(2 \mid e_H) = p(4 \mid e_H) = 0.5$. (Naturally, $p(0 \mid e_H) = 0$.) The worker has utility function given by $U(y, e) = \ln(y) - e$, where y represents the income earned by the worker and e the level of effort exerted. Suppose the worker can generate 1 unit of income with certainty just by watching TV at home. Watching TV requires no effort; as a result, the reservation utility of the worker is $\underline{U} = \ln(1) = 0$.

   a) Show that *any* wage contract $\{w(x_i)\}_{i=1,2,3}$ such that: $\{w(0) = 0, w(2) > 0, w(4) > 0\}$ and $0.5\left[\ln(w(2)) + \ln(w(4))\right] - e_H \geq 0$, induces the worker to exert high effort.

   b) Comment on the following assertion: "the *optimal* contract inducing high effort provides *full* insurance to the worker". (Hint: don't attempt to answer just out of the top of your head – state the problem and solve for the optimal contract that generates $e = e_H$.)

   c) How can you reconcile your answer to the previous question with the results shown in Section 3.3 of Macho-Stadler and Pérez-Castrillo. In particular, third paragraph in page 44 therein starts by saying "Since $\mu > 0$, the agent's wage [required to induce e_H] varies according to the result obtained. In particular, the wage will be greater the smaller is the ratio p_i^L/p_i^H."

3. Solve Mas-Colell, Whinston & Green 14.B.3 (a) and (b).

4. Suppose there exists 1 bank in the economy with the monopoly over a total amount of loanable funds equal to £50. The opportunity cost for liquid capital is equal to 1 (imagine the bank can lend at no risk to the government at *net* real interest rate equal to zero). In addition to the bank, there exist also 45 (potential) entrepreneurs who own zero initial wealth. Each (potential) entrepreneur may invest in one out of two different projects: a or b. Both a and b require 1 unit of initial capital outlay, which individuals must then borrow from the bank. Investment projects are risky; denoting by Y_s the output generated by project s, we have:

$$Y_a = \begin{cases} 2 & \text{with probability } P_a = 0.7 \\ 0 & \text{with probability } 1 - P_a. \end{cases}$$

$$Y_b = \begin{cases} 7 & \text{with probability } P_b = 0.1 \\ 0 & \text{with probability } 1 - P_b. \end{cases}$$

   a) Which project would (potential) entrepreneurs prefer to invest in? Does it depend on the interest rate, R, charged by the bank?

   b) What is the optimal contract offered by the bank, assuming that the bank cannot control/observe whether borrowers invest in project a or b? Is there credit rationing in equilibrium?