

## Appendix C: Online Appendix

This appendix extends the main results of our benchmark model in Section 3, when using a more general probability density function  $g(\cdot)$  from where each sectoral productivity parameter  $\eta_{z,c} \in \mathbb{Z} \times \{H, F\}$  is independently drawn. All relevant proofs can be found at the end of the appendix.

### Setup and Environment

As in Section 3 of the paper, we consider a setting with two economies:  $H$  and  $F$ . In each economy, there is a unit continuum of individuals, each of them endowed with one unit of labour time. Labour is immobile across countries. We take the wage in  $H$  as the numeraire, by setting  $w_H = 1$ , and denote by  $w_F$  the wage in  $F$ . All consumers in the world share identical preferences, as described in Section 2. Both  $H$  and  $F$  are open to international trade, and we assume there exist no trade cost. In country  $c \in \{H, F\}$  there exist a continuum of firms that may transform  $(1 + \eta_{z,c}q) / \kappa$  units of labour into one unit of commodity  $(z, q)$ , where the parameter  $\kappa \geq 1$  denotes the common level of total factor productivity. Heterogeneities in sectoral productivity across countries arise from the fact that each  $\eta_{z,c} \in \mathbb{Z} \times \{H, F\}$  is *independently* drawn from a probability density function  $g(\cdot)$  with full support over  $\mathbb{R}_{++}$ . As in the main text of the paper, we henceforth drop the subscript  $z$  whenever doing so generates no confusion.

### Equilibrium and Specialization along the Growth Path

Using the notation introduced in Definition 1, we can write down the budget constraint for an individual from country  $c \in \{H, F\}$  as follows:

$$\begin{aligned}
 & \int_0^{\hat{\eta}_H^c} \frac{(1 + \eta_H)}{\kappa} \left[ 1 - G\left(\frac{1 + \eta_H}{w_F} - 1\right) \right] dG(\eta_H) \\
 & \quad + \alpha \int_{\hat{\eta}_H^c}^{\infty} \frac{1}{\kappa} (1 - G(\hat{\eta}_F^c)) dG(\eta_H) \\
 & + w_F \left[ \int_0^{\hat{\eta}_F^c} \frac{(1 + \eta_F)}{\kappa} [1 - G((1 + \eta_F)w_F - 1)] dG(\eta_F) \right. \\
 & \quad \left. + (1 - \alpha) \int_{\hat{\eta}_F^c}^{\infty} \frac{1}{\kappa} (1 - G(\hat{\eta}_H^c)) dG(\eta_F) \right] = w_c;
 \end{aligned} \tag{1}$$

where  $\alpha = 1$  when  $w_F > 1$ ,  $\alpha = 0$  when  $w_F < 1$ , and  $\alpha \in [0, 1]$  when  $w_F = 1$ .

The general equilibrium requires that the trade balance between  $H$  and  $F$  is in equilibrium. Such equilibrium condition is given by the following equality:

$$\begin{aligned}
& \int_0^{\widehat{\eta}_H^F} \left( \frac{1 + \eta_H}{\kappa} \right) \left[ 1 - G \left( \frac{1 + \eta_H}{w_F} - 1 \right) \right] dG(\eta_H) \\
& \quad + \alpha \int_{\widehat{\eta}_H^F}^{\infty} \frac{1}{\kappa} \left( 1 - G(\widehat{\eta}_F^F) \right) dG(\eta_H) \\
& = w_F \left[ \int_0^{\widehat{\eta}_F^H} \left( \frac{1 + \eta_F}{\kappa} \right) [1 - G((1 + \eta_F) w_F - 1)] dG(\eta_F) \right. \\
& \quad \left. + (1 - \alpha) \int_{\widehat{\eta}_F^H}^{\infty} \frac{1}{\kappa} \left( 1 - G(\widehat{\eta}_H^H) \right) dG(\eta_F) \right]. \tag{2}
\end{aligned}$$

The LHS of (2) equals the total value of exports by  $H$  to  $F$ , while the RHS of (2) equals the total value of imports by  $H$  from  $F$ .

The following lemma briefly characterizes some important qualitative effects of changes of  $\kappa$  and  $w_F$  on the quality thresholds as introduced by Definition 1 in the main text of the paper.

**Lemma 1** *Let  $c, c' \in \{H, F\}$ . If, in equilibrium,  $\widehat{\eta}_{c'}^c$  is finite, then: i)  $\partial \widehat{\eta}_{c'}^c / \partial \kappa > 0$ ; ii)  $\partial \widehat{\eta}_{c'}^H / \partial w_F < 0$ ; iii)  $\partial \widehat{\eta}_{c'}^F / \partial w_F > 0$ .*

The next lemma deals with the specific case in which wages are the same in  $H$  and  $F$ .

**Lemma 2** *If, in equilibrium,  $w_F = 1$ , then:  $\widehat{\eta}_H^H = \widehat{\eta}_F^H = \widehat{\eta}_H^F = \widehat{\eta}_F^F = \widehat{\eta}$ . In addition, the threshold function is given by:*

$$\widehat{\eta} = \begin{cases} \widehat{\eta}(\kappa) & \text{if } 1 \leq \kappa \leq 1 + 2(E[\eta] - \phi), \\ \infty & \text{if } \kappa > 1 + 2(E[\eta] - \phi); \end{cases} \tag{3}$$

where:  $\phi \equiv \int_0^\infty \eta g(\eta) G(\eta) d\eta$ , hence  $0 < \phi < E[\eta]$ . Also,  $\eta(1) = 0$  and  $\partial \eta(\kappa) / \partial \kappa > 0$  for all  $1 \leq \kappa < 1 + 2(E[\eta] - \phi)$ .

We can now determine the equilibrium (relative) wage in this symmetric two-country world economy.

**Proposition 1** *In equilibrium,  $w_F = 1$  for any feasible level of  $\kappa$ .*

Proposition 1 shows that, in this symmetric two-country world economy, the relative wage remains unchanged and equal to unity all along the growth path. This also implies that the real income in both  $H$  and  $F$  is always equal to their common  $\kappa$ .

Along the growth path, economies also experience changes in their productive structure at the sectoral level. In particular, our model predicts that richer economies will tend to buy a larger fraction of their consumption goods from producers who display a comparative advantage in those goods (that is, from producers with relatively low realizations of  $\eta$ ). We define the average degree of comparative advantage of producers catering to country  $c$ 's demand as follows:

$$E[\eta^c | \text{demanded}] \equiv 2 \int_0^{\widehat{\eta}(\kappa)} \eta [1 - G(\eta)] dG(\eta) + \int_{\widehat{\eta}(\kappa)}^{\infty} \eta [1 - G(\widehat{\eta}(\kappa))] dG(\eta); \quad (4)$$

A lower value of (4) means that consumers are buying from producers displaying (on average) a lower  $\eta$ .

**Proposition 2** *The level of  $E[\eta^c | \text{demanded}]$  is non-increasing in  $\kappa$ .*

*i) for all  $1 \leq \kappa \leq 1 + 2(E[\eta] - \phi)$ ,  $E[\eta^c | \text{demanded}]$  is strictly decreasing in  $\kappa$ , with its highest value  $E[\eta^c | \text{demanded}] = E[\eta]$  when  $\kappa = 1$ ;*

*ii) for all  $\kappa > 1 + 2(E[\eta] - \phi)$ ,  $E[\eta^c | \text{demanded}] = 2(E[\eta] - \phi)$ , where  $\phi$  was previously defined in Lemma 2.*

Proposition 2 refers to the average value of  $\eta$  calculated over the *entire* consumption bundle purchased by a consumer (either from  $H$  or from  $F$ ), disregarding the origin of the producer selling the commodities. We can extend that result to the average value of  $\eta$  taking into account only *imported* goods purchased by the consumer. The average degree of comparative advantage of *exporters* catering to country  $c$ 's *imported* demand can be written as follows:

$$E[\eta^c | \text{imported}] \equiv \frac{\int_0^{\widehat{\eta}(\kappa)} \eta [1 - G(\eta)] dG(\eta) + \frac{1}{2} \int_{\widehat{\eta}(\kappa)}^{\infty} \eta [1 - G(\widehat{\eta}(\kappa))] dG(\eta)}{\int_0^{\widehat{\eta}(\kappa)} [1 - G(\eta)] dG(\eta) + \frac{1}{2} \int_{\widehat{\eta}(\kappa)}^{\infty} [1 - G(\widehat{\eta}(\kappa))] dG(\eta)}. \quad (5)$$

Noting that  $\int_0^{\widehat{\eta}(\kappa)} [1 - G(\eta)] dG(\eta) + \frac{1}{2} \int_{\widehat{\eta}(\kappa)}^{\infty} [1 - G(\widehat{\eta}(\kappa))] dG(\eta) = 1/2$ , it follows that  $E[\eta^c | \text{imported}] = E[\eta^c | \text{demanded}]$ . As a result, all the statements in Proposition (2) identically apply to imported goods.

A related result can be found when comparing the comparative advantage of exporters (measured again by the relative level of  $\eta$ ) in cases of exports in high quality versus exports in low quality. Essentially, the widening of heterogeneities in labour productivity at the upper level of qualities implies that exporters of goods in  $q = 1$  will exhibit (on average) a higher productivity than those exporting their products in  $q = 0$ . We define below the degree of

comparative advantage of exporters from country  $c'$ , conditional on the level of quality of their exports to country  $c$ .

$$E [\eta_{c'}^{\text{expo}} | q = 1] = \frac{\int_0^{\hat{\eta}^c} \eta_{c'} [1 - G(\eta_{c'})] dG(\eta_{c'})}{G(\hat{\eta}^c) - [G(\hat{\eta}^c)]^2 / 2}, \quad (6)$$

$$E [\eta_{c'}^{\text{expo}} | q = 0] = \frac{\int_{\hat{\eta}^c}^{\infty} \eta_{c'} dG(\eta_{c'})}{1 - G(\hat{\eta}^c)}; \quad (7)$$

where (6) and (7) are already taking into account that  $\hat{\eta}_{c'}^c = \hat{\eta}_c^c = \hat{\eta}^c$  when  $w_F = 1$ . From (6) and (7), the following proposition straightforwardly obtains

**Proposition 3** *The average degree of comparative advantage of exporters from country  $c'$  is increasing in the level of quality of their exports. That is,*

$$E [\eta_{c'}^{\text{expo}} | q = 1] < E [\eta_{c'}^{\text{expo}} | q = 0], \quad \text{where } c' \in \{H, F\}.$$

## Proofs

**Proof of Lemma 1.** Preliminarily, notice that: a) for a given  $c \in \{H, F\}$ ,  $d\hat{\eta}_F^c/d\kappa \lesseqgtr 0 \Leftrightarrow d\hat{\eta}_H^c/d\kappa \lesseqgtr 0$  and  $d\hat{\eta}_F^c/dw_F \lesseqgtr 0 \Leftrightarrow d\hat{\eta}_H^c/dw_F \lesseqgtr 0$ ; b)  $\partial\alpha/\partial\kappa = 0$  and  $\partial\alpha/\partial w_F \geq 0$ . Let  $c = H$ .

The proof for  $c = F$  follows an analogous reasoning, and it is available upon request.

**Part i)** Compute total differentiation of (1) with respect to  $\hat{\eta}_H^H$  and  $\kappa$ :

$$\begin{aligned} & (1 + \hat{\eta}_H^H) \left[ 1 - G(\hat{\eta}_F^H) \right] g(\hat{\eta}_H^H) d\hat{\eta}_H^H - \alpha \left[ 1 - G(\hat{\eta}_F^H) \right] g(\hat{\eta}_H^H) d\hat{\eta}_H^H \\ & - \alpha w_F^{-1} \left[ 1 - G(\hat{\eta}_F^H) \right] g(\hat{\eta}_F^H) d\hat{\eta}_H^H + (1 + \hat{\eta}_F^H) \left[ 1 - G(\hat{\eta}_H^H) \right] g(\hat{\eta}_F^H) d\hat{\eta}_H^H \\ & - (1 - \alpha) w_F \left[ 1 - G(\hat{\eta}_F^H) \right] g(\hat{\eta}_H^H) d\hat{\eta}_H^H - (1 - \alpha) \left[ 1 - G(\hat{\eta}_H^H) \right] g(\hat{\eta}_F^H) d\hat{\eta}_H^H = d\kappa. \end{aligned}$$

Rearranging terms and simplifying:

$$\begin{aligned} d\kappa = & \left\{ \left[ 1 + \hat{\eta}_H^H - \alpha - (1 - \alpha) w_F \right] \left[ 1 - G(\hat{\eta}_F^H) \right] g(\hat{\eta}_H^H) \right. \\ & \left. + \left[ 1 + \hat{\eta}_F^H - (\alpha/w_F + 1 - \alpha) \right] \left[ 1 - G(\hat{\eta}_H^H) \right] g(\hat{\eta}_F^H) \right\} d\hat{\eta}_H^H. \end{aligned}$$

Denote the RHS of this last expression by  $\Phi d\widehat{\eta}_H^H$ . Notice now that, from the definition of  $\alpha$ , we have  $\alpha + (1 - \alpha) w_F \leq 1$  and  $\alpha/w_F + 1 - \alpha \leq 1$ , hence:

$$\begin{aligned}\Phi &= \left[1 + \widehat{\eta}_H^H - \alpha - (1 - \alpha) w_F\right] \left[1 - G\left(\widehat{\eta}_F^H\right)\right] g\left(\widehat{\eta}_H^H\right) \\ &\quad + \left[1 + \widehat{\eta}_F^H - \alpha/w_F - (1 - \alpha)\right] \left[1 - G\left(\widehat{\eta}_H^H\right)\right] g\left(\widehat{\eta}_F^H\right) \\ &\geq \widehat{\eta}_H^H \left[1 - G\left(\widehat{\eta}_F^H\right)\right] g\left(\widehat{\eta}_H^H\right) + \widehat{\eta}_F^H \left[1 - G\left(\widehat{\eta}_H^H\right)\right] g\left(\widehat{\eta}_F^H\right) > 0.\end{aligned}$$

Hence,  $d\widehat{\eta}_H^H/d\kappa > 0$ . Our first preliminary statement then implies that  $d\widehat{\eta}_F^H/d\kappa > 0$  also holds.

**Part ii)** Compute total differentiation of (1) with respect to  $\widehat{\eta}_H^H$  and  $w_F$ :

$$\begin{aligned}\Phi d\widehat{\eta}_H^H &+ \left\{ \int_0^{\widehat{\eta}_H^H} \left(\frac{1 + \eta_H}{w_F}\right)^2 g\left(\frac{1 + \eta_H}{w_F} - 1\right) dG(\eta_H) \right. \\ &\quad + \mathcal{I}_\alpha \int_{\widehat{\eta}_H^H}^\infty \left[1 - G\left(\widehat{\eta}_F^H\right)\right] dG(\eta_H) + \int_0^{\widehat{\eta}_F^H} (1 + \eta_F) [1 - G((1 + \eta_F) w_F - 1)] dG(\eta_F) \\ &\quad - w_F \int_0^{\widehat{\eta}_F^H} (1 + \eta_F)^2 g((1 + \eta_F) w_F - 1) dG(\eta_F) - \mathcal{I}_\alpha \int_{\widehat{\eta}_F^H}^\infty w_F [1 - G\left(\widehat{\eta}_H^H\right)] dG(\eta_F) \\ &\quad \left. + (1 - \alpha) \int_{\widehat{\eta}_F^H}^\infty [1 - G\left(\widehat{\eta}_H^H\right)] dG(\eta_F) \right\} dw_F = 0;\end{aligned}$$

where  $\mathcal{I}_\alpha \in [0, 1]$  takes a non-zero value only when the relative wage is in the vicinity of  $w_F = 1$ . Note that the integral in the fifth term can be rewritten on  $\eta_H$  by considering the following relationship:

$$\eta_F = \phi(\eta_H) \equiv \frac{1 + \eta_H}{w_F} - 1, \text{ with } d\eta_F = \frac{1}{w_F} d\eta_H; \quad (8)$$

hence we can rewrite the fifth term as:

$$\begin{aligned}w_F \int_0^{\widehat{\eta}_F^H} (1 + \eta_F)^2 g((1 + \eta_F) w_F - 1) dG(\eta_F) &= \\ &= w_F \int_0^{(1 + \widehat{\eta}_F^H) w_F - 1} \left(\frac{1 + \eta_H}{w_F}\right)^2 g(\eta_H) g\left(\frac{1 + \eta_H}{w_F} - 1\right) \frac{1}{w_F} d\eta_H = \\ &= \int_0^{\widehat{\eta}_H^H} \left(\frac{1 + \eta_H}{w_F}\right)^2 g\left(\frac{1 + \eta_H}{w_F} - 1\right) dG(\eta_H); \end{aligned}$$

which then cancels out with the second term. Notice that the third and sixth terms also cancel out, as they have non-zero value only when in the vicinity of  $w_F = 1$ , exactly the case when the difference in their magnitudes is infinitesimal and therefore negligible. Thus, simplifying and rearranging, we obtain:

$$\frac{d\widehat{\eta}_H^H}{dw_F} = \frac{\int_0^{\widehat{\eta}_F^H} (1 + \eta_F) [1 - G((1 + \eta_F) w_F - 1)] dG(\eta_F) + (1 - \alpha) \int_{\widehat{\eta}_F^H}^\infty (1 - G(\widehat{\eta}_H^H)) dG(\eta_F)}{-\Phi}.$$

Thus, bearing in mind that  $\Phi > 0$ ,  $d\hat{\eta}_H^H/dw_F < 0$  must hold. Finally, once again, our first preliminary statement then implies that  $d\hat{\eta}_F^H/dw_F < 0$  also holds. ■

**Proof of Lemma 2.** Notice first that, when  $w_F = 1$ ,  $\hat{\eta}_F^H = \hat{\eta}_H^H = \hat{\eta}^H$  and  $\hat{\eta}_F^F = \hat{\eta}_H^F = \hat{\eta}^F$ . As a consequence, bearing in mind that for all  $\eta_H = \eta_F$ , we have  $G(\eta_H) = G(\eta_F)$ , the budget constraints for individuals from  $H$  and  $F$  may be written, respectively, as follows:

$$\begin{aligned} 2 \int_0^{\hat{\eta}^H} (1 + \eta_H) [1 - G(\eta_H)] dG(\eta_H) + \left(1 - G(\hat{\eta}^H)\right)^2 &= \kappa, \\ 2 \int_0^{\hat{\eta}^F} (1 + \eta_H) [1 - G(\eta_H)] dG(\eta_H) + \left(1 - G(\hat{\eta}^F)\right)^2 &= \kappa. \end{aligned}$$

Hence, the two equations above imply  $\hat{\eta}^H = \hat{\eta}^F = \hat{\eta}$ .

Next, the expressions in (3) may be derived by writing down the budget constraint as follows

$$2 \int_0^{\hat{\eta}} (1 + \eta) [1 - G(\eta)] g(\eta) d\eta + (1 - G(\hat{\eta}))^2 = \kappa.$$

Bearing in mind that  $\int_0^{\hat{\eta}} G(\eta) g(\eta) d\eta = [G(\hat{\eta})]^2 / 2$  and  $\int_0^{\hat{\eta}} g(\eta) d\eta = G(\hat{\eta})$ , the above equation simplifies to:

$$2 \int_0^{\hat{\eta}} \eta [1 - G(\eta)] g(\eta) d\eta = \kappa - 1. \quad (9)$$

Differentiating (9), we get that:

$$\frac{d\hat{\eta}}{d\kappa} = [2\hat{\eta} [1 - G(\hat{\eta})] g(\hat{\eta})]^{-1};$$

which is strictly positive for any  $\hat{\eta} < \infty$ . Finally, by noting that  $\phi \equiv \int_0^\infty \eta g(\eta) G(\eta) d\eta < \int_0^\infty \eta g(\eta) d\eta = E[\eta]$ , we may obtain the condition  $\hat{\eta} = \infty$  for all  $\kappa > 1 + 2(E[\eta] - \phi)$ . ■

**Proof of Proposition 1.** Firstly, notice from Lemma 2 that, when  $w_F = 1$ , we have  $\hat{\eta}_H^H = \hat{\eta}_F^H = \hat{\eta}_H^F = \hat{\eta}_F^F = \hat{\eta}$ . In addition, notice  $G(\eta_H) = G(\eta_F)$  for any  $\eta_H = \eta_F$ . As a result, it turns out that when  $w_F = 1$  the first terms of both sides of (2) are equivalent, namely

$$\int_0^{\hat{\eta}(\kappa)} (1 + \eta_H) [1 - G(\eta_H)] dG(\eta_H) = \int_0^{\hat{\eta}(\kappa)} (1 + \eta_F) (1 - G(\eta_F)) dG(\eta_F).$$

Therefore, as long as  $\alpha = 1/2$ ,  $w_F = 1$  is an equilibrium for any  $\kappa \geq 1$ .

Secondly, suppose in equilibrium  $w_F > 1$ . In this case we have  $\alpha = 1$ . As a consequence, the equilibrium in the labour markets of  $H$  and  $F$  would read, respectively, as follows:

$$\begin{aligned} \int_0^{\hat{\eta}_H^H} \frac{1 + \eta_H}{\kappa} [1 - G(\phi(\eta_H))] dG(\eta_H) + \int_{\hat{\eta}_H^H}^1 \frac{1}{\kappa} [1 - G(\hat{\eta}_F^H)] dG(\eta_H) \\ + \int_0^{\hat{\eta}_H^F} \frac{1 + \eta_H}{\kappa} [1 - G(\phi(\eta_H))] dG(\eta_H) + \int_{\hat{\eta}_H^F}^1 \frac{1}{\kappa} [1 - G(\hat{\eta}_F^F)] dG(\eta_H) = 1; \end{aligned} \quad (10)$$

and

$$\begin{aligned} \int_0^{\widehat{\eta}_F^H} \frac{1 + \eta_F}{\kappa} [1 - G(\phi^{-1}(\eta_F))] dG(\eta_F) \\ + \int_0^{\widehat{\eta}_F^F} \frac{1 + \eta_F}{\kappa} [1 - G(\phi^{-1}(\eta_F))] dG(\eta_F) = 1; \end{aligned} \quad (11)$$

where recall that  $\phi(\eta_H)$  was defined in (8). Notice now that, when  $w_F > 1$ ,  $\widehat{\eta}_H^H > \widehat{\eta}_F^H$  and  $\widehat{\eta}_H^F > \widehat{\eta}_F^F$ . Hence, from (10) and (11) it must be the case that

$$\begin{aligned} \int_0^{\widehat{\eta}_F^H} (1 + \eta) [1 - G(\phi(\eta))] dG(\eta) + \int_0^{\widehat{\eta}_F^F} (1 + \eta) [1 - G(\phi(\eta))] dG(\eta) \\ < \int_0^{\widehat{\eta}_F^H} (1 + \eta) [1 - G(\phi^{-1}(\eta))] dG(\eta) + \int_0^{\widehat{\eta}_F^F} (1 + \eta) [1 - G(\phi^{-1}(\eta))] dG(\eta). \end{aligned}$$

However this is impossible when  $w_F > 1$ , since in that case  $\phi(\eta) = (1 + \eta)/w_F - 1 < \phi^{-1}(\eta) = (1 + \eta)w_F - 1$ .

Lastly, suppose in equilibrium  $w_F < 1$ . In this case we have  $\alpha = 0$ , and the equilibrium in the labour markets of  $H$  and  $F$  would be given, respectively, by:

$$\begin{aligned} \int_0^{\widehat{\eta}_H^H} \frac{1 + \eta_H}{\kappa} [1 - G(\phi(\eta_H))] dG(\eta_H) \\ + \int_0^{\widehat{\eta}_H^F} \frac{1 + \eta_H}{\kappa} [1 - G(\phi(\eta_H))] dG(\eta_H) = 1; \end{aligned} \quad (12)$$

$$\begin{aligned} \int_0^{\widehat{\eta}_F^H} \frac{1 + \eta_F}{\kappa} [1 - G(\phi^{-1}(\eta_F))] dG(\eta_F) + \int_{\widehat{\eta}_F^H}^1 \frac{1}{\kappa} [1 - G(\widehat{\eta}_H^H)] dG(\eta_H) \\ + \int_0^{\widehat{\eta}_F^F} \frac{1 + \eta_F}{\kappa} [1 - G(\phi^{-1}(\eta_F))] dG(\eta_F) + \int_{\widehat{\eta}_F^F}^1 \frac{1}{\kappa} [1 - G(\widehat{\eta}_H^F)] dG(\eta_H) = 1. \end{aligned} \quad (13)$$

Notice now that, when  $w_F < 1$ ,  $\widehat{\eta}_H^H < \widehat{\eta}_F^H$  and  $\widehat{\eta}_H^F < \widehat{\eta}_F^F$ . Hence, from (12) and (13) it must be the case that

$$\begin{aligned} \int_0^{\widehat{\eta}_F^H} (1 + \eta) [1 - G(\phi(\eta))] dG(\eta) + \int_0^{\widehat{\eta}_F^F} (1 + \eta) [1 - G(\phi(\eta))] dG(\eta) \\ > \int_0^{\widehat{\eta}_F^H} (1 + \eta) [1 - G(\phi^{-1}(\eta))] dG(\eta) + \int_0^{\widehat{\eta}_F^F} (1 + \eta) [1 - G(\phi^{-1}(\eta))] dG(\eta). \end{aligned}$$

However this is impossible when  $w_F < 1$ , since in that case  $\phi(\eta) = (1 + \eta)/w_F - 1 > \phi^{-1}(\eta) = (1 + \eta)w_F - 1$ . ■

**Proof of Proposition 2.** Differentiating (4) with respect to  $\kappa$  yields:

$$\frac{\partial E[\eta^c | \text{demanded}]}{\partial \kappa} = \widehat{\eta}(\kappa) [1 - G(\widehat{\eta}(\kappa))] g(\widehat{\eta}(\kappa)) \frac{\partial \widehat{\eta}}{\partial \kappa} - g(\widehat{\eta}(\kappa)) \frac{\partial \widehat{\eta}}{\partial \kappa} \int_{\widehat{\eta}(\kappa)}^{\infty} \eta g(\eta) d\eta.$$

Notice that  $\int_{\widehat{\eta}(\kappa)}^{\infty} \eta g(\eta) d\eta = [1 - G(\widehat{\eta}(\kappa))] E[\eta | \eta > \widehat{\eta}(\kappa)]$ ; hence, the above expression can be re-written as:

$$\frac{\partial E[\eta^c | \text{demanded}]}{\partial \kappa} = (\widehat{\eta}(\kappa) - E[\eta | \eta > \widehat{\eta}(\kappa)]) [1 - G(\widehat{\eta}(\kappa))] g(\widehat{\eta}(\kappa)) \frac{\partial \widehat{\eta}}{\partial \kappa}.$$

As a result, whenever  $\partial \widehat{\eta} / \partial \kappa > 0$ , it must be the case that  $\partial E[\eta^c | \text{dem.}] / \partial \kappa < 0$ , since  $\widehat{\eta}(\kappa) < E[\eta | \eta > \widehat{\eta}(\kappa)]$ . The other two parts of the proof follow immediately from replacing, respectively,  $\widehat{\eta}(1) = 0$  and  $\widehat{\eta}(\kappa) = \infty$  for all  $\kappa > 1 + 2(E[\eta] - \phi)$ , into (4). ■

**Proof of Proposition 3.** Firstly, it should be noted that  $\int_0^{\widehat{\eta}^c} \eta_{c'} [1 - G(\eta_{c'})] dG(\eta_{c'}) < \widehat{\eta}^c \int_0^{\widehat{\eta}^c} [1 - G(\eta_{c'})] dG(\eta_{c'}) = \widehat{\eta}^c [G(\widehat{\eta}^c) - [G(\widehat{\eta}^c)]^2 / 2]$ ; and also that  $\int_{\widehat{\eta}^c}^{\infty} \eta_{c'} dG(\eta_{c'}) > \widehat{\eta}^c \int_{\widehat{\eta}^c}^{\infty} dG(\eta_{c'}) = \widehat{\eta}^c [1 - G(\widehat{\eta}^c)]$ ; then, it immediately follows that  $E[\eta_{c'}^{\text{expo}} | q = 1] < \widehat{\eta}^c < E[\eta_{c'}^{\text{expo}} | q = 0]$ . ■