Final Exam 2010

Allievi and Master in Economics Programmes

1. Suppose a large number of firms compete for risk-neutral workers in the labour market. Workers are heterogenous in terms of their intrinsic ability θ, where $1 \leq \theta \leq 2$. Ability is subject to private information. Ability is distributed in the population of workers according to the following *cumulative* distribution function: $F(\theta) = \theta - 1$ for all $\theta \in [1, 2]$; in other words, θ is *uniformly* distributed along the interval $[1, 2]$. Besides working for firms, workers have access to an outside option that yields income $r(\theta) = 1 + \frac{1}{4}\theta$.

   a) What is the Pareto-efficient allocation of workers to tasks in the economy? **5 points**

   b) Calculate and plot the conditional expected productivity $E[\theta \mid \theta \in \Theta(w)]$, and find the competitive equilibrium/equilibria in the labour market. Discuss whether the equilibrium (equilibria) is (are) Pareto-efficient. **15 points**

   c) Suppose now that $r(\theta) = 1 + \alpha\theta$, where $0 < \alpha < \frac{1}{4}$. How does this case differ from the previous point? (you may solve this case just graphically) **5 points**

2. Consider the following principal-agent contractual problem. The principal is risk-neutral and must choose whether to hire the agent for a specific task and design the optimal contract to offer given the informational constraints. The agent may exert three different levels of effort, ordered by increasing intensity: $e = \{0, 1, 2\}$. The agent's effort is unobservable to the principal. Output, which is publicly observable, may take three possible values: $y = \{0, 10, 20\}$. The probabilities of each level of output, conditional on the effort exerted by the agent, are as follows:

effort level	$y = 0$	$y = 10$	$y = 20$
$e = 0$	1	0	0
$e = 1$	0	0.8	0.2
$e = 2$	0	0.2	0.8

(1)

The agent is risk-averse and also dislikes exerting effort. In particular, his preferences are summarised by the following utility function:

$$u(w, e) = \begin{cases} \sqrt{w} - e & \text{if } w \geq 0 \\ -\infty & \text{if } w < 0 \end{cases},$$

Lastly, the agent has access to a reservation utility $\underline{u} = 0$.

a) Solve the optimisation problem faced by the principal. Which is the profit-maximising contract offered by the principal to the agent? **15 points**

b) What would be the profit-maximising contract offered if effort were publicly observable? Compare it to the previous contract – are they different or identical? (Explain why). **10 points**

3. Take the adverse selection model *à la* Rothschild & Stiglitz (1976) presented during the lectures. Assume the following:

- There exist two types of workers: high- and low-ability workers.
- The fractions of high- and low-ability in the population are the same; that is: $\frac{1}{2}$ of workers are of high ability, and $\frac{1}{2}$ are of low ability.
- High-ability workers generate output $x_s = 100$ with probability p, while they produce output $x_f = 25$ with probability $1 - p$, where $0 < p < 1$.
- Low-ability workers generate output $x_f = 25$ with probability 1.
- Workers are risk averse, with (Bernoulli) utility of income given by: $u(w) = \ln(w)$.

i) Under which conditions may a (competitive) equilibrium exist in this setup? **20 points**

ii) Could you give some interpretation for the above result? **5 points**

4. Consider the signaling model with two types of workers presented in Mas-Colell *et al*, section 13.C. Assume that $\theta_L = 0$ and $\theta_H = 1$, and both types of workers have access to the same outside option: $r(\theta_L) = r(\theta_H) < 0$. Workers may acquire education $e \geq 0$ (measured in years of schooling) before entering the labour market. Education does not affect workers' (innate) productivity. Let the utility function of low-type and high-type agents be, respectively: $u(w, e \mid \theta_L) = w - e$ and $u(w, e \mid \theta_H) = w - \frac{1}{2}e$. Denote by $\mu(\theta_H \mid e)$ the belief by firms that someone with e years of schooling is of type θ_H. Assess whether the following statement is true or false, justifying your answer:

"There exists a *separating* PBE sustained by beliefs $\mu(\theta_H \mid e)$, where those beliefs satisfy the following conditions: *i*) $e < \mu(\theta_H \mid e) < 1$ for all $0 < e < 1$; *ii*) $\mu(\theta_H \mid e) = 1$ for all $e \geq 1$; *iii*) and $\mu(\theta_H \mid e) = 0$ for $e = 0$". **25 points**