

Final Exam 2009 – Allievi Programme

1. Suppose a large number of firms compete for risk-neutral workers in the labour market. Workers are heterogenous in terms of their intrinsic ability $\theta \in [0, 1]$. Ability is subject to private information (i.e., only each worker knows his own ability). Ability is distributed in the whole population of workers according to the following *cumulative* distribution function: $F(\theta) = \theta$ for all $\theta \in [0, 1]$; in other words, ability of workers is *uniformly* distributed along the interval $[0, 1]$. Besides working for firms, workers have access to an outside option (say, growing maize at their backyards) that yields income $r(\theta) = \alpha\theta^2$, where $0 < \alpha \leq 1$.
 - a) Is it always Pareto-efficient that *all* workers end up working for firms? (Justify your answer). If your previous answer was 'no', clarify for which values of α (if any) it is actually Pareto-efficient that all workers work for firms. **7 points**
 - b) Take the specific case in which $\alpha = 1$. Plot the conditional expected productivity schedule, $E[\theta \mid \theta \in \Theta(w)]$, against w , and find the competitive equilibrium/equilibria therein. Discuss whether the equilibrium (equilibria) is (are) Pareto-efficient. **15 points**
 - c) For which values of $\alpha \in (0, 1]$ would the economy be able to reach a Pareto-efficient competitive equilibrium? **8 points**

2. Consider the contracting problem faced by a landowner and a landless labourer. The landowner is risk-neutral and chooses whether to hire the labourer to work in his fields. The labourer may shirk or work hard, which we denote, respectively, by $e = \{1, 2\}$. Output, which is random, may take two possible values, $y = \{5, 20\}$, and is (partially) affected by effort. The probabilities of each level of output, conditional on effort by the labourer, are as follows:

effort level	$y = 5$	$y = 20$
$e = 1$	0.7	0.3
$e = 2$	0.2	0.8

(1)

The landless labourer is risk-averse and also dislikes working hard on the fields. In particular, his preferences are summarised by the following utility function:

$$u(w, e) = \begin{cases} \sqrt{w} - e & \text{if } w \geq 0 \\ -\infty & \text{if } w < 0 \end{cases},$$

Lastly, the labourer has access to a reservation utility $\underline{u} = 0$.

Assume effort is unobservable:

- a) Solve the optimisation problem faced by the landowner. Write down the profit-maximising contract that the landowner offers to the labourer. **15 points**
- b) Suppose now there are three effort levels, $e = \{1, 2, 3\}$. Assume that $\Pr(y = 20 \mid e = 3) = 1$ (that is, if $e = 3$, high output is produced for sure). For the previous $e = 1$ and $e = 2$, Table (1) still represents their output probabilities. Which is now the profit-maximising wage contract offered by the landowner to the labourer? Does this contract provide full insurance? **15 points**

3. Adverse selection is generally thought to be quite pervasive in insurance markets. This has led many economists to argue for the introduction of compulsory insurance schemes as a (second-best) solution to the market inefficiencies engendered by adverse selection.

Suppose we are dealing here with the health insurance market. In particular, and to simplify matters, suppose we are analysing the case of insurance against the treatment costs of heart attacks. If someone suffers from a heart attack, he must receive a treatment that costs him $C > 0$ (measured in money). Assume there exist two types of people: those at high risk of suffering from a heart attack and those at low risk of it. The probability of heart attack for high-risk guys is given by p_G , while for low-risk guys is p_L , where $0 < p_L < p_G < 1$. Suppose that risk factors are subject to private information. The insurance market is perfectly competitive (hence, it offers contracts *à la Rothschild & Stiglitz*). All individuals have initial wealth equal to $W > C$ and are risk-averse with the same utility function $u(Y)$, where Y denotes ex-post wealth.

Suppose the Ministry of Health is pondering whether or not to force *everybody* in the economy to buy a compulsory health insurance against heart attacks at the actuarially fair price. Will this policy be always Pareto-improving (in terms of insurance provision) relative to the competitive market provision of insurance against heart attacks? If your answer is 'no', under what conditions will the policy be Pareto-improving?

Hint: For writing down the insurance contracts, you can assume that individual i must pay (ex ante) a prime R_i for each unit of insurance he buys. Each unit of insurance pays back (ex post) as indemnity 1 unit of money in the event of 'suffering from a heart attack', and zero otherwise. Hence, if you denote by $q_i \geq 0$ the units of insurance that individual i decides to buy, his expected utility reads as follows:

$$E(U_i) = p_i u(W - C - R_i q_i + q_i) + (1 - p_i) u(W - R_i q_i);$$

where p_i denotes the probability of heart attack for individual i .

You are **NOT** expected to work out an algebraic solution. A careful graphic representation of the problem, alongside a rigorous verbal justification for your answer, is more than enough. **20 points**

4. Consider the signaling model with two types of workers presented in Mas-Colell *et al*, section 13.C. Assume that $\theta_L = 2$ and $\theta_H = 4$. Workers can acquire education ε before entering the labour market. There are three possible levels of education that individuals may choose to get, namely: $\varepsilon = \{0, 1, 2\}$; which denote no education, primary school, and high school, respectively. Education does not affect workers' (innate) productivity. Let the utility function for each type of agents be: $u(w, \varepsilon | \theta_L) = w - 2\varepsilon^2$ and $u(w, \varepsilon | \theta_H) = w - \varepsilon^2$. Assume all types have reservation utility given by $r(\theta) = 0$.

i) Which levels of education may be sustained as separating Perfect Bayesian Equilibrium? **10 points**

ii) Assume the fraction of types- H is larger than the fraction of types- L . Suppose the government decides to abolish education in the economy. The government declares "we decided to eliminate the education system because this is Pareto-improving". Is that correct? **10 points**