Dynamic Optimization for Economics

Final (full) Examination

June 13, 2024

Please, answer at least 5 of the following 6 questions. ***Time allowed****: two hours and 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.*

Question 1. Continuity of functions on metric spaces: Definition, properties and characterization(s). Upper semicontuinuity: Definition and relations to continuity, with (at least graphical) examples.

Question 2. Compact sets. Definition, properties and characterization(s). Recall at least one fundamental result which uses compactness. Prove that if $A \subseteq B$ and B is compact, A is compact iff it is closed.

Question 3. The contraction mapping theorem. Statement and proof. State (and prove if time allows) a set of conditions on an operator $T : B(S) \to B(S)$ that guarantee that T be a contraction (Blackwell's condition).

Question 4. Separating hyperplane theorems in $\mathbb{R}^n$. Briefly present the main ideas involved, and then state and prove at least one result involving separation of *two sets*.

Question 5. Upper and lower hemi-continuity. Definitions and examples. Relations with other properties (closed- , compact- and convex-valued, closed- and convex-graph). The maximum theorem.

Finally, consider the correspondence $\varphi(x) : [0, 3] \to \mathbb{R}$ defined by

$$\varphi(x) = \begin{cases} [1,2] & x \in [0,1] \\ \{3/2\} & x \in (1,2] \\ [1,2] & x \in (2,3] \end{cases}$$

Identify all the properties it satisfies.

Question 6. Describe and define a finite-horizon dynamic programming problem. Discuss, in as much detail as possible, the structure of its optimal strategies (under natural assumptions on the structure of the problem), and how to find them.