

# Mathematical Economics

Final (full) Examination

July 7, 2023

Please, answer at least 5 of the following 6 questions. **Time allowed:** two hours and 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.

**Question 1.** Metric spaces, normed vector spaces, and inner product spaces. Definitions, properties and examples. In particular, discuss (and compare) the cases of the spaces  $\ell_2$  and  $\ell_\infty$ .

**Question 2.** Totally bounded sets. Definition, properties and characterization(s). If  $A \subset B \subset M$  and if  $B$  is totally bounded in the metric space  $M$ , show that  $A$  is totally bounded.

**Question 3.** Pointwise and uniform convergence of functions. Definitions and examples. Is there a connection between uniform convergence and convergence in a specific norm? Results about properties of uniform limits and their consequences for spaces of functions.

**Question 4.** Separating hyperplane theorems in  $\mathbb{R}^n$ . Briefly present the main ideas involved, and then state and prove at least one result involving separation of *two sets*.

**Question 5.** Upper and lower hemi-continuity. Definitions and examples. Relations with other properties (closed-, compact- and convex-valued, closed- and convex-graph). The maximum theorem.

Finally, consider the correspondence  $\varphi(x) : [0, 3] \rightarrow \mathbb{R}$  defined by

$$\varphi(x) = \begin{cases} [0, 1] & x \in [0, 1) \\ \{1\} & x \in [1, 2) \\ [1, 2] & x \in [2, 3] \end{cases}$$

Identify all the properties it satisfies.

**Question 6.** Describe and define a finite-horizon dynamic programming problem. Discuss, in as much detail as possible, the structure of its optimal strategies (under natural assumptions on the structure of the problem), and how to find them.