

# Mathematical Economics

Final Examination

July 7, 2017

Please, answer at least 5 of the following 6 questions. **Time allowed:** two hours 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.

**Question 1.** Metric spaces and normed vector spaces. Definitions, properties and examples. In particular, define and discuss the example of the space  $\ell_2$ .

**Question 2.** Continuity of functions. Definition, properties and characterization(s).

**Question 3.** Pointwise and uniform convergence of functions. Definitions and examples. What is the connection between uniform convergence and convergence in a specific norm? Results about properties of uniform limits and their consequences for spaces of functions.

**Question 4.** Upper and lower hemi-continuity. Definitions and examples. Relations with other properties (closed-, compact- and convex-valued, closed- and convex-graph). The maximum theorem.

Finally, consider the correspondence  $\varphi(x) : [0, 3] \rightarrow \mathbb{R}$  defined by

$$\varphi(x) = \begin{cases} [0, 1] & x \in [0, 1) \\ [0.25, 0.75] \cup \{1\} & x \in [1, 2) \\ \{0.5\} & x \in [2, 3] \end{cases}$$

Identify all the properties it satisfies.

**Question 5.** Articulate a finite-horizon dynamic programming problem, and discuss, in as much detail as possible, the structure of its optimal strategies (under natural assumptions on the structure of the problem).

**Question 6.** Articulate a stationary discounted dynamic programming (SDDP) problem and define an optimal strategy for such problem. After recalling the important property that the value function  $V$  must satisfy, present, in as much detail as possible (including its mathematical background), the result which shows that an optimal strategy must induce a revenue function ( $w(\sigma)(s)$ ) that satisfies such property. State clearly which assumptions on the SDDP you are making.