Mathematical Economics

Final Examination

June 7, 2017

Please, answer at least 5 of the following 6 questions. ***Time allowed****: two hours 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.*

Question 1. The Weierstrass theorem in its two variants, with appropriate definitions and examples. Is the function $f(x) = -(x^4 + 2x)$ coercive?

Question 2. The contraction mapping theorem. Statement and proof. State (and prove if time allows) a set of conditions on an operator $T : B(S) \to B(S)$ that guarantee that T be a contraction (Blackwell's condition).

Question 3. Separating hyperplane theorems in $\mathbb{R}^n$. Briefly present the main ideas involved, and then state and prove at least one such result.

Question 4. Upper and lower hemi-continuity. Definitions and examples. Relations with other properties (closed- , compact- and convex-valued, closed- and convex-graph). The maximum theorem.

Finally, consider the correspondence $\varphi(x) : [0, 3] \to \mathbb{R}$ defined by

$$\varphi(x) = \begin{cases} [0, 1] & x \in [0, 1] \\ [0.25, 0.75] & x \in (1, 2) \\ [0, 1] & x \in [2, 3] \end{cases}$$

Identify all the properties it satisfies.

Question 5. Articulate a finite-horizon dynamic programming problem, and discuss, in as much detail as possible, the structure of its optimal strategies (under natural assumptions on the structure of the problem).

Question 6. Articulate a stationary discounted dynamic programming (SDDP) problem and define an optimal strategy for such problem. Discuss the first basic assumption on the structure of the problem and explain, in as much detail as possible, which pathologies it is meant to exclude. Finally, after having defined the value function for the problem, recall the important property that the value function has to satisfy given the assumption discussed earlier, proving what you can. What is the role played by this property in the solution of SDDPs?