DECISION THEORY: HOMEWORK # 2

1. Solve problem 5 of chapter 3 in Kreps. While you are at it, think about this (for extra credit): The statement in Theorem 3.6 is actually false. Can you find a counterexample to it?

2. Let $\succeq$ be a relation on $X \subseteq \mathbf{R}^n$. Suppose that $\succeq$ is represented by a continuous utility function $u : X \to \mathbf{R}$. Show that then $\succeq$ is a continuous weak order. That is, $\succeq$ is complete and transitive, and it satisfies the following continuity property:

   Definition 1 *The relation $\succeq$ on $X \subseteq \mathbf{R}^n$ is* continuous *if for every $x, y \in X$: If $x \succ y$ and $\{x_n\}_{i=1}^n$ is a sequence such that $x_n \in X$ and $x_n \to x$, then there is N large enough so that for all $n \geq N$, $x_n \succ y$. If $y \succ x$ and $\{x_n\}_{i=1}^n$ is a sequence such that $x_n \in X$ and $x_n \to x$, then there is N large enough so that for all $n \geq N$, $y \succ x_n$.*

3. Something that I "forgot to do" in class: Prove that if X is a (not necessarily finite) subset of $\mathbf{R}^n$, $\mathcal{B}$ is a collection of budget sets such that each B is nonempty and compact, and the weak order $\succeq$ is continuous (in the sense above), then for every B, $C^*(B, \succeq) \neq \emptyset$. Thus, the choice structure $(\mathcal{B}, C^*)$ will satisfy WARP. [Remark: You actually do not need X to be a subset of $\mathbf{R}^n$ for this result.]