Decision Theory: Homework # 1

1. In class, I claimed that a cardinal utility is necessary for some decision criteria to make sense. Referring to the hand-out by Luce and Raiffa, look at the minimax risk criterion, the Hurwicz α-pessimism criterion, and the criterion based on the principle of insufficient reason. For each of these criteria, find an example of a decision problem such that, if we take a monotonic increasing transformation of the utility numbers, we change the prescription of the decision rule.

2. Solve exercise 1 of Chapter 1 in Kreps.

3. Solve exercise 2 of Chapter 2 in Kreps.

4. Prove the following statement from class: Suppose that $\sim$ is an equivalence relation (i.e., a symmetric, reflexive and transitive relation) on some set X. Define $X/\sim$ to be the set of the collections of indifferent elements in X. That is,

   $$X/\sim \equiv \{A \subseteq X : A = \{y \in X : y \sim x\}, \exists x \in X\}.$$

   Show that $X/\sim$ is a *partition* of X. That is, for all $A, B \in X/\sim$, $A \cap B = \emptyset$, and $\cup_{A \in X/\sim} A = X$.

   Indeed, a type of converse of the statement is also true. That is, given any reflexive binary relation $\sim$, the set $X/\sim$ is a partition *only if* $\sim$ is symmetric, and transitive. Prove it.

   Would a plain converse be true? That is, is the following statement true: given any binary relation $\sim$, the set $X/\sim$ is a partition *only if* $\sim$ is reflexive, symmetric, and transitive? Clearly it's not, otherwise I wouldn't ask. Provide a counterexample.