Games and Decisions: Final Exam

January 11, 2013

Please, answer the following questions. The total number of points is 120. ***Time allowed****: two hours 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.*

1. (35 points) Let $\mathcal{P}_S$ be the set of the simple (with finite support) probability distributions on a prize set X.

(a) (8 points) Begin by stating what it means for a preference relation $\succeq$ on $\mathcal{P}_S$ to be representable by expected utility (EU). That is: "there is ... such that ..."

(b) (5 points) Two functions u and v are both utilities for $\succeq$ when...?

(c) (5 points) When do we say that $\succeq$ is *risk averse*? (I mean the definition in terms of preferences.)

(d) (7 points) What property of the utility function u corresponds to risk aversion?

(e) (10 points) Assuming that u is twice differentiable if necessary, prove that such property still holds after you transform the utility as described in point b of this question.

2. (25 points) After stating them, show that axioms NM1, NM3 (i.e., axioms 5.1 and 5.3 in Kreps) and the following property do *not* imply NM2 (5.2 in Kreps). The property is: For every $p, q, r \in \mathcal{P}_S$ and every $\alpha \in [0, 1]$, if $p \sim q$ then $\alpha p \oplus (1 - \alpha)r \sim \alpha q \oplus (1 - \alpha)r$.

3. (20 points) Consider a finite normal form game $G = (A_1, ..., A_n, u_1, ..., u_n)$, where A_i denotes player i's set of actions and $u_i : A_1 \times ... \times A_n \to \mathbb{R}$ denotes his payoff function. Consider another game $G' = (A_1, ..., A_n, v_1, ..., v_n)$. The two games G and G' have the same players and the same set of actions for each player. Furthermore, there exist two real numbers α and β such that for every player $i = 1, ..., n$ and for every action profile $a \in A_1 \times ... \times A_n$ the following equality holds:

$$v_i (a) = \alpha u_i (a) + \beta$$

Find the values of α and β such that the two games have exactly the same set of Nash equilibria. Provide a formal proof for your answer.

4. (40 points) A jury of three individuals has to decide whether to acquit (A) or convict (C) a defendant. There are two states of the world: the defendant can be either innocent (I) or guilty (G). Each juror has some private information about the state. Specifically, each juror observes either an innocent signal i or a guilty signal g (the signal is the juror's type). All profiles of signals (types) are equally likely:

$$\Pr(iii) = \Pr(iig) = \ldots = \Pr(ggg) = \frac{1}{8}.$$

Suppose that the defendant is guilty if and only if at least two jurors observe a guilty signal. Notice that this assumption implies that if the jurors could observe all the signals, then they would know the state of the world (however each juror observes only her own signal).

All the jurors have identical payoffs u which depend on the verdict, A or C, and the state, I or C. Specifically:

$$u(A, I) = 1, \quad u(C, G) = 1, \quad u(A, G) = 0, \quad u(C, I) = 0.$$

The jurors choose a verdict by voting. Each juror can vote in favor of conviction (c) or acquittal (a). Under majority rule the defendant is convicted if at least two jurors choose c. Under the unanimity rule the defendant is convicted if all the jurors vote in favor of c.

Notice that a voting rule specifies a Bayesian game. In fact, given a profile a votes (actions) and types (signals) we can compute the verdict and the state and, thus, the players' payoffs. For example, suppose the profile of types is (iig) and the profile of votes is (acc). Since there is only one guilty signal the defendant is innocent. Moreover, the defendant is convicted under the majority rule and acquitted under the unanimity rule. If we let π and v denote a player's payoff under the majority and the unanimity rule, respectively, we have:

$$\pi((acc), (iig)) = u(C, I) = 0,$$

$$v((acc), (iig)) = u(A, I) = 1.$$

We say that a juror votes "sincerely" if she chooses a when her type is i and c when her type is g. Sincere voting denotes the strategy profile in which each player votes sincerely.

(a) Consider the majority rule. Is sincere voting a Bayesian Nash equilibrium?

(b) Show that under the unanimity rule sincere voting is not a Bayesian Nash equilibrium.

(c) For this question restrict attention to symmetric pure strategy profiles in which both types of the jurors use the same action. Find all Bayesian Nash equilibria under the majority rule and the unanimity rule.