Games and Decisions: Final Exam

December 18, 2012

Please, answer the following questions. The total number of points is 120. ***Time allowed****: two hours 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.*

1. (30 points) A senior faculty member, Professor T** P**, tries to maximize the mean quality of his colleagues. He is voting on the choice between two candidates. One, Prof. A, will be a tenured appointment, and her quality is, in his assessment 8.5 (more correctly: this is his von Neumann-Morgenstern index for having her around). The other candidate, Doctor B, is for a nontenured position. He believes that there is a 20% chance that B will turn out to be a 20, 70% that he will be a 6 and 10% that he will be a 2. Suppose that Prof. P** expects that another candidate, Doctor C, will be hired in the case that B is denied tenure six years hence, and his value is certainly 7 (Prof. P** has great foresight abilities, and he knows that Prof. A will not be available any longer then.) Finally assume that P**'s time preference are time additive and they assign equal weight to division quality in the next six years (B's tenure track, if he were hired) and to quality thereafter.

(a) (8 points) If he would later be able to make an unconstrained tenure decision, should Prof. P** vote now for candidate A (tenured) or B (untenured)?

(b) (8 points) If the tenure process is instead a rubber stamp, so that nobody can be denied tenure, should he vote for A or B?

(c) (8 points) Assume that internal candidates have a certain advantage in the tenure process, so that only candidates with quality strictly less than 3 can be turned down, and all others are tenured. How should Prof. P** vote?

(d) (6 points) How would your answers to all the previous questions change if the distribution of B's quality were replaced by its mean?

2. (30 points) In as much detail as you can, discuss the uniqueness results in the utility representation theorems:

(a) (15 points) in the case of choice under certainty (i.e., when the preference relation $\succcurlyeq$ is over the set X of the final outcomes)

(b) (15 points) in the case of choice over lotteries (i.e., $\succcurlyeq$ is defined over the set $\mathcal{P}_S$ of the lotteries over X)

In particular, explain the practical difference between such uniqueness properties when talking about the utility $u(x)$ of a given outcome x.

3. (30 points) Consider the following game. $n \geq 2$ players try to guess what $2/3$ of the average of their guesses will be. That is, each player $i = 1, ..., n$ chooses a number $x_i \in [0, 1]$. The players make their choices simultaneously. Player i wins the game if x_i is closer to $\frac{2}{3}\left(\frac{x_1+x_2+...+x_n}{n}\right)$ than any other number x_j, $j \neq i$. The prize is equal to one and is split equally among all the winners of the game (the losers get zero).

a) Write the normal-form representation of this game.

b) Show that there is a unique strategy profile which survives the process of iterative removal of weakly dominated strategies.

c) Show that the game admits a unique Nash equilibrium and find it (for this part, you can restrict attention to pure strategies).

4. (30 points) Consider a finite normal form game $(S_1, ..., S_n, u_1, ..., u_n)$. Let $\Sigma_i = \Delta(S_i)$ denote player i's set of mixed strategies with typical element denoted by σ_i.

a) Show that if

$$u(s_i, s_{-i}) \geq u(s_j, s_{-i})$$

for all $s_{-i} \in \times_{j \neq i} S_j$, then

$$u(s_i, \sigma_{-i}) \geq u(s_j, \sigma_{-i})$$

for all $\sigma_{-i} \in \times_{j \neq i} \Sigma_j$.

b) Show that if s_i and s_i' are two best responses to a certain strategy profile s_{-i}, then any randomization between s_i and s_i' is also a best response to s_{-i}.

c) Suppose that s_i and s_i' are two best responses to a certain strategy profile s_{-i}. Can you conclude that a mixed strategy which assigns positive probability to s_i'', with $s_i'' \neq s_i$ and $s_i'' \neq s_i'$, is not a best response to s_{-i}?