Games and Decisions: Take-Home Final Exam

DUE: Monday March 7 at 12 noon

Rules of the Game On your honor, you are bound to abide by the following rules:

1. The exam is to be taken in 48 consecutive hours, whenever and wherever you see fit. However, the exam has to be turned in by the deadline of 12 noon on Monday, **without any exception**.
2. The exam is open-book and open notes.
3. But you are not allowed to consult directly or indirectly any other source (including of course living beings)!
4. Please feel free to send me e-mail if you have any questions about procedure (no hints given, however!).

The total number of points is 200. Points for each question are indicated. Remember to follow the dominant strategy: *If you get stuck in one problem, jump immediately to some other one, and get back only when you have done everything which you find easy.*

Most importantly: **Have fun!**

1. (60 points) An oil wildcatter must decide whether or not to drill at a given site before his option expires. His acts are to drill (f_1) or not to drill (f_2). He is uncertain whether the hole is dry (state ω_1), wet (state ω_2) or soaking (state ω_3). If he decides to drill, he will have to face a cost of $70,000 for drilling, and his expectation of the (gross) revenue from extraction is $ 0 if the hole is dry, $ 120,000 if the hole is wet, and 270,000 if the hole is soaking. (Clearly, if he doesn't drill he pays nothing and receives nothing in every state.)

   At a cost of $ 10,000, our wildcatter could take seismic soundings (experiment E_1) which will help determine the underlying geological structure at the site. The soundings will disclose whether the terrain below has *no structure* (outcome NS) — that's bad, or *open structure* (outcome OS) — that's so so, or *closed structure* (outcome CS) — that's really hopeful. The experts have kindly provided us with Table 1, which shows the joint probabilities of states and seismic outcomes (and their respective marginals):

state	NS	OS	CS	marg.
ω_1	.300	.150	.050	.500
ω_2	.090	.120	.090	.300
ω_3	.020	.080	.100	.200
marg.	.410	.350	.240	1.000

   Assume that the wildcatter maximizes EU with an affine utility (i.e., is risk-neutral).

   (a) (10 points) What is his optimal choice without experimentation?

   (b) (15 points) Call "perfect information" that corresponding to being told exactly what the real state of the world is. I want you calculate the *expected monetary value of perfect information*. That is, assume that you could perform an experiment E_2 that (differently from experiment E_1 above) reveals exactly what the state of the world is: what would be the maximal amount of money you would pay in order to run this experiment (the sum that would make you *indifferent* between running E_2 and taking your optimal choice without experimentation)?

   (c) (20 points) Draw a decision tree for this problem and devise the optimal strategy for experimentation and action in this problem.[1] [HINT: Solve backwards]

   (d) (15 points) What is the *expected value of the seismic information*? That is, what would be the maximal sum that you would be willing to spend to run experiment E_1 and know its result (which does not have to be $ 10,000, mind)?

2. (40 points) A small "diversion" on the uniqueness of EU preferences. Given two EU preferences $\succsim_1$ and $\succsim_2$, say that a consequence $x \in \mathcal{X}$ is *jointly essential for the orderings* $\succsim_1$ *and* $\succsim_2$ if there are $\overline{x}, \underline{x} \in \mathcal{X}$ such that $\delta_{\overline{x}} \succ_i \delta_x \succ_i \delta_{\underline{x}}$

[1] Assume that only experiment E_1 can be performed or no experiment at all — that you may want to denote E_0; the experiment E_2 discussed above is not factually possible.

for $i = 1, 2$ (notice that this condition implies that the two orderings must at least agree on the ranking of these three lotteries). Now prove the following result:

Proposition 1 *Let $\succsim_1$ and $\succsim_2$ be EU orderings on $\mathcal{P}$. Suppose that there is a consequence $x \in \mathcal{X}$, jointly essential for $\succsim_1$ and $\succsim_2$, such that, for all $\alpha \in [0,1]$ and all $x', x'' \in \mathcal{X}$,*

$$\delta_x \sim_1 \alpha\, \delta_{x'} + (1-\alpha)\, \delta_{x''} \Longrightarrow \delta_x \sim_2 \alpha\, \delta_{x'} + (1-\alpha)\, \delta_{x''}.$$

Then $\succsim_1 = \succsim_2$.

This shows that an EU ordering is uniquely defined by much less than an indifference curve: It is defined by its ranking of a fixed consequence and all the binary lotteries.

3. (50 points) Compute all Nash equilibria of the following normal-form game.

	L	C	R
U	$3, 1$	$-1, 0$	$4, 9$
M	$2, 3$	$4, 2$	$8, 6$
D	$5, -2$	$-3, -5$	$2, -7$

4. (50 points) Consider the following Cournot oligopoly model. There are two firms. Let q_i denote the quantity produced by firm $i = 1, 2$. The market clearing price, p, depends on the total output:

$$p(q_1, q_2) = \begin{cases} 12 - 2(q_1 + q_2) & \text{if } 12 - 2(q_1 + q_2) \geqslant 0, \\ 0 & \text{otherwise.} \end{cases}$$

The two firms have zero fixed costs and constant marginal costs. The marginal cost c_i of firm $i = 1, 2$ is private information. In particular, c_i can be either $c_H = 4$ or $c_L = 1$. The probability distribution of the marginal costs is given by:

$$\text{Prob}(c_L, c_L) = \text{Prob}(c_L, c_H) = \text{Prob}(c_H, c_L) = \tfrac{1}{3} \quad \text{Prob}(c_H, c_H) = 0.$$

Compute the symmetric (pure-strategy) Bayesian Nash equilibrium of the game.