DECISIONS AND UNCERTAINTY: HOMEWORK # 5

1. Imagine a world in which the result of any lottery depends only on one event A happening or not. The probability of event A is $p \in (0,1)$. Thus every simple act f can be written as an ordered pair (f_1, f_2), where f_1 is the (monetary) result to be obtained if A occurs, and f_2 is the result obtained otherwise. Until instructed otherwise, suppose that the set of acts is all of $\mathbf{R}^2$, and that a DM satisfies all the von Neumann-Morgenstern axioms (including axiom NM0) on it, and that her utility function $u : \mathbf{R} \to \mathbf{R}$ is continuous and increasing.
   - (a) Draw a set of coordinate axis (this is called the *Hirshleifer-Yaari diagram*). Depict the 45° line. Give an intuitive interpretation of the points on that line.
   - (b) Suppose that the DM is risk-neutral. Using the graph drawn in (a), draw a typical indifference locus, that is, choose a point f and depict graphically the set of all the acts which are indifferent to it. Explain your picture analytically. Considering the point f' at which the indifference locus cuts the 45° line, what is the value of its slope at f'? Draw another indifference locus, corresponding to higher expected utility.
   - (c) Suppose now that the DM is risk-averse. Draw a typical indifference locus, again explaining analytically your picture. Again, what is the slope of the indifference locus when it cuts the 45° line?
   - (d) For a given nondegenerate act f, draw its indifference locus and depict graphically its certainty equivalent $c(f)$.
   - (e) How would the previous answers change if: (i) u could be discontinuous, (ii) u could be nondecreasing?
   - (f) Imagine now that the DM's preferences can be represented by a RDEU functional, with a strictly convex distortion function φ. Start by assuming that the DM has a linear utility function (for instance $u(x) = x$), as in Yaari's dual

EU model. Depict the typical indifference locus, with particular attention to what happens when it crosses the $45°$ line.

(g) Define, analogously to what we did for expected utility, the certainty equivalent of act f to be a certain amount which is indifferent to f. Explain why, under the assumptions we made so far, it is unique, so that we can label it $c(f)$. Depict it graphically in the diagram. Now define the risk premium of f as usual:

$$\pi(f) = E(f) - c(f),$$

depict it graphically (as an act!). This is another way to make a point we made in class. Which?

(h) Suppose now that we make u strictly concave, what is going to happen to the risk premium of a given act f?

(i) Let x be a given (positive, let's say) amount, and consider the class of all the acts of the form $f(t) = x + t\epsilon$, where $t \geq 0$ and ϵ is a nondegenerate r.v. such that $E\epsilon = 0$. Write $\pi(t) \equiv \pi(f(t))$. It is immediate to notice that $\pi(t) \to 0$ as $t \to 0$ and $\pi(0) = 0$. *Graphically* for the case of a DM with concave u, study the behavior of $\pi(t)$ as $t \to 0$ when the DM has EU preferences, and when she has RDEU preferences (with strictly concave φ). Both obviously go to zero, but they do it qualitatively in a different fashion. Explain what I mean by this. This different behavior has been formally denominated as follows: EU preferences display have *second order* risk aversion, whereas RDEU preferences display *first order* risk aversion (any guess why?).

2. In the Anscombe and Aumann model of Chapter 7 of Kreps, consider the following dominance axiom:

   Axiom 7.17 *Suppose that $h, h' \in H$ are such that $h_s \succeq h'_s$ for every $s \in S$, then $h \succeq h'$.*

   Show that it can be substituted for axiom 7.16 in Theorem 7.17. That is, under axioms 7.1, 7.2, 7.3 and 7.14, axioms 7.16 and 7.17 are equivalent.

3. Solve problem 6 of Chapter 7 in Kreps.