

## DECISIONS AND UNCERTAINTY: HOMEWORK # 4

1. Prove that an increasing utility function has *constant absolute risk aversion* (i.e., it is such that  $\lambda(x) = -u''(x)/u'(x) = K$  for some  $K \in \mathbf{R}$ ) *if and only if*
  - $(K \neq 0) \ u(x) = -ae^{-Kx} + b$ , with  $a, b \in \mathbf{R}$
  - $(K = 0) \ u(x) = ax + b$ , with  $a > 0$  and  $b \in \mathbf{R}$ .
2. Solve exercise 7 of chapter 6 in Kreps.
3. Solve problem 13 of Chapter 6 in Kreps.
4. You are just back from your first consulting session with a pension fund manager. This manager has to decide among three possible long-term investment plans, which we will label  $f, g, h$ . After some work you have managed to extract the following information from her: The relevant state space is  $S = \{s_1, \dots, s_7\}$ , and her beliefs can be represented by the probability measure  $P$  (which is the second line in the table below). Moreover she believes that the behavior of  $f, g$  and  $h$ , as functions on  $S$  is reflected in the table below. She wants to be consistent with the Subjective Expected Utility model.

	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$	$s_7$
$P$	0.05	0.05	0.1	0.1	0.2	0.2	0.3
$f$	0	0	-100	200	-50	100	0
$g$	100	200	-100	200	0	50	-50
$h$	100	200	-50	-100	0	0	100

- (a) Calculate the distribution functions induced by  $f, g, h$  on the prize space  $C = \{-100, -50, 0, 50, 100, 150, 200\}$ .
- (b) Suppose that you have elicited the utility function of the fund manager, and estimated that it behaves roughly as  $u(x) = \sqrt{x + 200}$ . What is her optimal choice?

Suppose, instead, that you do not know the manager's utility function (HINT: Recall results we briefly saw that link stochastic dominance with unanimity of EU preferences with utility functions with specific properties).

- (c) Are there options which your manager will *not* pick if she is greedy (the more money the better)?
- (d) What if you know that she is risk averse, but you can't tell that she likes more money than less (for instance you might suspect that she is only interested in having a certain target return, and not more). Can you rule some plan out?
- (e) Finally what if she is both greedy and risk averse?