

Decisions and Uncertainty: Final Exam

January 18, 2024

Please, answer the following questions. The total number of points is 120. **Time allowed:** two hours and 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.

1. (30 points) The representation of preferences under certainty.
 - (a) Describe the “binary relation” approach to representing such preferences. State the axiom or axioms, discuss it/them briefly, and then state and sketch a proof of a representation theorem.
 - (b) Discuss the difference between dealing with a finite or countable consequence set and an uncountable one, in as much detail as you can. In particular, discuss the lexicographic counterexample (no proof needed), and what type of further assumption on preferences is needed to exclude it.
 - (c) What axiom of Expected Utility plays the role of the assumption that you discussed in the previous item? State it and provide an interpretation.

2. (30 points) Consider the problem of choice under risk when consequences are monetary amounts; i.e. $C = [a, b]$.
 - (a) Define and discuss the notion of *first-order stochastic dominance*, providing an example and the graphical intuition.
 - (b) Define and discuss the notion of *second-order stochastic dominance*, providing some graphical intuition. What is the relationship between first- and second-order stochastic dominance?
 - (c) Define and discuss the notion of *riskiness à la Rothschild-Stiglitz*. What is the relationship between Rothschild-Stiglitz riskiness and second-order stochastic dominance? What is the relationship between Rothschild-Stiglitz riskiness and first-order stochastic dominance?
 - (d) Assuming that you are considering DMs who satisfy the EU model, how can you characterize these three orderings over lotteries (in terms of u functions)?

3. (30 points) The subjective expected utility (SEU) model of Savage.
- (a) (9 points) Describe in as much detail as you can the construction of the subjective probability charge.
 - (b) (6 points) Limiting yourself to simple acts, start by describing the mathematical representation of preferences in the Savage model, and recalling its uniqueness properties and the properties of the (subjective) probability P .
 - (c) (8 points) Then describe briefly the axioms that imply such representation (always for simple acts). In particular, discuss axioms P2-P4 and axiom P6, and their role for the representation. Why is Axiom P5 important?
 - (d) (7 points) Provide as much detail as you can on how we make use of the von Neumann-Morgenstern model in the proof of the representation.
4. (30 points) Kreps' preference for flexibility model and its "descendants":
- (a) Discuss in detail Kreps' model: the framework, the axioms, and the representation theorem (for extra credits prove the representation theorem).
 - (b) Discuss the Gul-Pesendorfer (GP) model and the Dekel-Lipman-Rustichini (DLR) model, and their relationship to the axioms in Kreps (i.e. which violation of Kreps' axioms motivated each of them).
 - (c) Discuss the TWO cases generating a preference for commitment in the GP model.
 - (d) Write a numerical example of the DLR model for which $F \subset G$ (strict containment) and $F \sim G$.