

Decisions and Uncertainty: Final Exam

December 19, 2023

Please, answer the following questions. The total number of points is 120. **Time allowed:** two hours and 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.

1. (30 points) The Expected Utility (EU) model of von Neumann and Morgenstern.
 - (a) Start by stating the EU axioms, and briefly comment on the interpretation of the Independence axiom, and on the role played by the Archimedean axiom.
 - (b) Next, state the representation theorem, and provide a sketch of the proof (what are the main steps?), without getting lost into too much detail. In the finite C case, what is the utility of a consequence x ?
 - (c) Finally, recall briefly what criticism has been addressed to the EU model, in particular focusing on the Independence axiom (notice: I do not want the precise details here, only the main ideas).
2. (30 points) Let \mathcal{P}_S be the set of the simple (with finite support) probability distributions on a prize set X .
 - (a) (4 points) When do we say that \succeq is *risk averse*? (I mean the definition in terms of preferences.) When is it *strictly* risk averse?
 - (b) (4 points) Given a preference which is represented by Expected Utility (see previous question), what property of the utility function u corresponds to risk aversion? Strict risk aversion? Illustrate the difference between the two properties!
 - (c) (7 points) Can you provide a graphical intuition (with a simple binary lottery) for this result?
 - (d) (8 points) Assuming that u is twice differentiable if necessary, prove that such property still holds after you take a positive affine transformation of the utility function.
 - (e) (7 points) Consider a bet that pays 10 euro if a regular die falls with 1-2 dots on top and 0 otherwise. Suppose that a decision maker prefers the bet to receiving 3.50 euros for sure. What can you say about her? Suppose that when the stakes are multiplied by 100 the decision maker

prefers 330 euros for sure to the bet. Does your earlier conclusion about her risk attitude still stand?

3. (30 points) Modelling subjective probability.
 - (a) Start by recalling the qualitative probability approach and its axioms. Discuss briefly: are the axioms necessary and/or sufficient for representation by a (finitely additive) probability?
 - (b) Savage's approach to subjective probabilities: What is added to what you recalled in the previous question? Discuss in as much detail as you can (be mindful about time!).
 - (c) Finally, describe the thought experiment known as the Ellsberg paradox, and explain (analytically) why it poses a problem for the development of subjective probability.

4. (30 points) The discounted utility model:
 - (a) Discuss the framework, the model and the axioms (in particular Stationarity).
 - (b) Write a condition on preferences (i.e., an axiom) that implies $\delta = 1$ in the geometric DU.
 - (c) What axiom of the GDU is violated if $\delta = 0$?
 - (d) Discuss the quasi-hyperbolic discounting model. For extra credits, try to find an axiom that would imply $\beta \geq 0.5$.
 - (e) Suppose that a quasi-hyperbolic discounter has to complete a task with the following profile of costs $(-2, -3, -5, -9, -19)$ meaning that, if the action is taken in period 1 its cost is (in utils) -2 , if taken in period 2 its cost is -3 , etc. Assuming that $\delta = 1$ and $\beta = 0.5$, describe the behavior of a naive and of a sophisticated agent."