Decisions and Uncertainty: Midterm Exam

November 4, 2022

Please, answer the following questions. The total number of points is 60. **Time allowed***: 90 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.*

1. (30 points) The Expected Utility (EU) model of von Neumann and Morgenstern.

   (a) Start by stating the EU axioms, and briefly comment on the interpretation of the Independence axiom and of the Archimedean axiom.

   (b) Next, state the representation theorem, and provide a sketch of the proof (what are the main steps?), without getting lost into too much detail. EXTRA CREDIT: What is the conceptual significance of the two mentioned axioms for the model?

   (c) What is the utility of a consequence x? Does this interpretation depend on the finiteness of the set of consequences C (motivate your answer)?

   (d) Finally, recall briefly what criticism has been addressed to the Independence axiom (notice: I do not want the precise details here, only the main idea/s).

2. (30 points) Consider the problem of choice under risk when consequences are monetary amounts.

   (a) Recall the definition of risk averse/neutral/seeking for a preference $\succcurlyeq$ over the set of probability distributions $\mathcal{P}$.

   (b) When $\succcurlyeq$ satisfies the EU axioms, which property of the representing utility u corresponds to risk aversion? Show this result graphically.

   (c) Consider now the following quadratic utility function $u(x) = x - kx^2$, for which values of k is the DM risk averse? Do you think that a utility function of this type makes always sense from the point of view of an economist, or that it does only on a subset of $\mathbb{R}$ (and in such case, which subset)? (HINT: Which assumption are economists happy making, when it comes to money?)

   (d) Suppose that the DM with the quadratic utility above is indifferent between a lottery that pays 12 and 5 with respective probabilities (1/2) and (1/2) and the certain amount 7. Can you say something about k?