Decisions and Uncertainty: Final Exam

December 16, 2021

Please, answer the following questions. The total number of points is 120. ***Time allowed****: two hours and 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.*

1. (30 points) The representation of preferences under certainty.

(a) Describe the "binary relation" approach to representing such preferences. State the axiom or axioms, discuss it/them briefly, and then state and sketch a proof of a representation theorem. Discuss the difference between dealing with a finite or countable consequence set and an uncountable one, in as much detail as you can.

(b) Limiting yourself to the finite C case, discuss the alternative approach of "revealed preferences," its main axiom and the relations between the two approaches.

2. (30 points) Let $\mathcal{P}_S$ be the set of the simple (with finite support) probability distributions on a prize set X.

(a) (7 points) Begin by stating what it means for a preference relation $\succeq$ on $\mathcal{P}_S$ to be representable by expected utility (EU). That is: "there is ... such that ..."

(b) (4 points) When do we say that $\succeq$ is *risk averse*? (I mean the definition in terms of preferences.)

(c) (4 points) What property of the utility function u corresponds to risk aversion?

(d) (8 points) Assuming that u is twice differentiable if necessary, prove that such property still holds after you a take a positive affine transformation of the utility function.

(e) (7 points) Consider a bet that pays 10 euro if a regular die falls with 1-2 dots on top and 0 otherwise. Suppose that a decision maker prefers the bet to receiving 3.50 euros for sure. What can you say about her? Suppose that when the stakes are multiplied by 100 the decision maker prefers 330 euros for sure to the bet. Does your earlier conclusion about her risk attitude still stand?

3. (30 points) The Anscombe-Aumann approach to modelling Expected Utility with subjective probabilities (i.e. Subjective Expected Utility):

(a) (5 points) Describe in as much detail as you can the decision setting that is used, and its mathematical properties.

(b) (10 points) State the axioms for the SEU model in this setting, and discuss them in as much detail as you can. Is there a different way to obtain state independence?

(c) (5 points) State the representation theorem (including uniqueness).

(d) (10 points) Finally, discuss the two alternative approaches to model randomized acts (mixtures), their differences and how Anscombe and Aumann tackle those differences.

4. (30 points) Discuss in detail the Kreps' model: the framework, the axioms, and the representation theorem (for extra credits prove the representation theorem). Discuss the Gul-Pesendorfer (GP) model and the Dekel-Lipman-Rustichini (DLR) model, and their relation to the axioms in Kreps (i.e. which violation of Kreps' axioms motivated each of them). Write two numerical examples in GP, one for which $x \succcurlyeq y$ and $x \cup y \sim y$, and one for which $x \succcurlyeq y$ and $x \cup y \succ y$. Write a numerical example of DLR for which $F \subset G$ (strict containment) and $F \sim G$.