Decisions and Uncertainty: Final Exam

January 13, 2017

Please, answer the following questions. The total number of points is 120. ***Time allowed:*** *two hours and 45 minutes. PLEASE PLEASE, make an effort to write in a legible and organized fashion.*

1. (40 points) In as much detail as you can, discuss the uniqueness results in the utility representation theorems:

(a) (15 points) in the case of choice under certainty (i.e., when the preference relation $\succcurlyeq$ is over the set C of the final outcomes)

(b) (15 points) in the case of choice over lotteries (i.e., $\succcurlyeq$ is defined over the set $\mathcal{P}_S$ of the lotteries over C)

Finally (10 points), explain the practical difference between such uniqueness properties when talking about the utility $u(x)$ of a given outcome $x \in C$.

2. (20 points) Suppose that a decision maker is indifferent between a bet, which pays 100 euro if some event A (e.g., "tomorrow the MIBTEL index will close up") obtains and 0 otherwise, and 49 euros. Assuming that this decision maker satisfies the subjective expected utility model, can you tell me what is her subjective probability $P(A)$? How would your answer change if I told you also that this decision maker's utility function is $u(x) = 2\sqrt{x} + 3$?

3. (30 points) The subjective expected utility (SEU) model of Savage.

(a) Describe in as much detail as you can the construction of the subjective probability charge.

(b) Limiting yourself to simple acts, start by describing the mathematical representation of preferences in the Savage model, and recalling its uniqueness properties and the properties of the (subjective) probability P.

(c) Then describe briefly the axioms that imply such representation (always for simple acts). In particular, discuss axioms P2 (the sure-thing principle), axiom P4 (the "payoff-independence" axiom) and axiom P6 (the "archimedean" axiom) and their role for the representation.

(d) Sketch briefly the steps of the proof of the representation theorem.

4. (30 points) The Ellsberg paradox and its consequences.

(a) Start by describing the thought experiment known as the Ellsberg paradox, and explain (analytically) why it poses a problem for the SEU model.

(b) In a famous experiment that I briefly described in class, Craig Fox and Amos Tversky asked people walking on the UC Berkeley campus (very close to San Francisco) to provide certainty equivalents for bets on the temperature in San Francisco and Istanbul. Specifically, they asked a set of subjects to state certainty equivalents for the two complementary bets that paid: a) \$ 100 if the temperature in San Francisco the following day was greater than or equal than 60 degrees Farenheit (\$ 0 otherwise), b) \$ 100 if the temperature in San Francisco the following day was less than 60 degrees Farenheit (\$ 0 otherwise). They then asked a set of subjects to do the same with analogous bets involving the temperature in Istanbul, and finally a third set of subjects to state certainty equivalents for all four possible bets. They found out what follows:

   i. those subjects that were asked to state certainty equivalents *only* for the bets in San Francisco on average gave certainty equivalents for the two bets that summed to \$ 39.89 (precisely, \$ 21.95 for the bet on the high temperature and \$ 17.94 for the bet on the low temperature),

   ii. those subjects that were asked to state certainty equivalents *only* for the bets in Istanbul on average gave certainty equivalents for the two bets that summed to \$ 38.37 (precisely, \$ 21.07 for the bet on the high temperature and \$ 17.29 for the bet on the low temperature),

   iii. those subjects that were asked to state certainty equivalents *for both sets of bets* on average gave certainty equivalents for the two San Francisco bets that summed to \$ 40.53 (=$22.74 + 17.79$) and for the two Istanbul bets that summed to \$ 24.69 (=$15.21 + 9.49$), almost 16 dollars less!

   Is the evidence provided by the group of subjects described in iii above in contrast with the SEU model? That is, can one propose probabilities for the relevant events and a utility function that would rationalize such choices? What about the subjects in groups i and ii? Do you notice anything strange about their preferences?