## Decisions and Uncertainty: Midterm Exam

November 18, 2016

*Please, answer the following questions. The total number of points is 60. Time allowed: two hours. PLEASE PLEASE, make an effort to write in a legible and organized fashion.* 

- 1. (35 points) Let  $\mathcal{P}_S$  be the set of the simple (with finite support) probability distributions on a prize set *X*.
  - (a) (4 points) Begin by stating what it means for a preference relation  $\succeq$  on  $\mathcal{P}_S$  to be representable by expected utility (EU). That is: "there is ... such that ..."
  - (b) (9 points) Recall briefly which axioms are key to obtaining such representation (no proof needed).
  - (c) (5 points) When do we say that  $\succeq$  is *risk averse*? (I mean the definition in terms of preferences.)
  - (d) (7 points) What property of the utility function *u* corresponds to risk aversion?
  - (e) (10 points) Assuming that *u* is twice differentiable if necessary, prove that such property still holds after you a take a positive affine transformation of the utility function.
- 2. (25 points) Suppose that the chief of "Protezione Civile" (the Italian agency that responds to natural catastrophes) is considering two possible criteria to decide whether or not to evacuate an area at risk of flooding whenever flooding is considered to be possible. The *ex ante* probability of flooding is assessed to be 1%. Intuitively, there are four possible outcomes to the choice being considered: (*a*) evacuation is not necessary, and it is not carried out, (*b*) an unneeded evacuation is carried out, (*c*) evacuation is carried out and there is flooding, (*d*) no evacuation is carried out, and there is flooding.

Suppose that the chief —who is a maximizer of expected utility– is indifferent between the certainty of *b* and a lottery of *a* with probability *p* and *d* with probability 1 - p, and also between the certainty of *c* and a lottery of *a* with probability *q* and *d* with probability 1 - q. Assume also that she prefers *a* to *d* and that  $p \in (0, 1)$  and  $q \in (0, 1)$ .

(a) (10 points) Construct a utility function for the chief.

- (b) (15 points) Suppose now that there are two criteria to decide whether to order an evacuation:
  - i. Criterion I prescribes an evacuation in 90% of the cases in which flooding does actually occur, and an unnecessary evacuation in 10% of the cases in which flooding does not occur.
  - ii. Criterion II prescribes an evacuation in 85% of the cases in which flooding does actually occur, and an unnecessary evacuation in 15% of the cases in which flooding does not occur.

Start by computing the probability distributions over final outcomes induced by each of the two criteria. Then, using the utility function that you obtained in the previous step, discuss which criterion should be preferred. Does your answer depend on p and q?