Game Theory, Spring 2021
Midterm Exam

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You have 90 minutes to complete this exam. Please answer the following three questions. Be sure to allocate your time in proportion to the points. Always justify your answers by providing a formal proof or a detailed argument. Good luck.

1. **[30 points]** Consider the following oligopoly model in which $n \geq 2$ firms compete by choosing their quantities $q_1, \ldots q_n$. The inverse demand function is $P(q_1 + \ldots + q_n) = \frac{a}{q_1 + \ldots + q_n}$, where a is a positive parameter. The cost of firm $i = 1, \ldots, n$ of producing the quantity q_i is cq_i^2, with $c > 0$. The payoff of each firm is equal to its profits.

   Construct the symmetric Nash equilibrium of the game.

2. **[35 points]** Consider the following all pay auction for an object. Three bidders, 1 and 2 and 3, simultaneously submit non-negative bids. The valuation of the object of bidder 1 is $v_1 = 5$, while the valuations of the remaining bidders are $v_2 = v_3 = 10$.

   The object is awarded to the bidder with the largest bid (ties are broken randomly, with equal probabilities) and each bidder pays his own bid (independently on whether he gets the good or not).

   If bidder i submits the bid b_i, his payoff is $v_i - b_i$ if he gets the good, and $-b_i$ if he does not get the good.

   Find a Nash equilibrium of the game.

   (Hint: Not all the bidders have to play a mixed strategy.)

3. **[35 points]** Consider a group of $n \geq 3$ students who work on a joint project. Each student can exert either a high or a low level of effort. The project is successful if at least two students exert high effort. If the project is successful (not successful) then each student obtains a reward equal to $v > 0$ (zero). The cost of exerting low effort is zero. Each student $i = 1, \ldots, n$ has private information about his cost c_i of exerting high effort. In particular, c_i is distributed uniformly over the interval $[0, v]$. Furthermore, the students' types are independent. The students choose their effort levels simultaneously. Finally, the payoff of each player is equal to the difference between the reward and his cost of effort.

   Construct a symmetric Bayesian Nash equilibrium.