Game Theory, Spring 2021
Final Exam

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You have two hours to complete this exam. Please answer the following three questions. Be sure to allocate your time in proportion to the points. Always justify your answers by providing a formal proof or a detailed argument. Good luck.

1. **[30 points]** A principal has the authority to make a decision. However, he can delegate the decision to an informed agent.

   There are two states of the world, 0 and 1. The two states are equally likely. If the principal makes the decision, then he chooses an action $a \in \mathbb{R}$ (without observing the state). If the principal delegates the final decision, then the agent chooses an action $a \in \mathbb{R}$ after observing the state.

   Suppose that the state is $\omega \in \{0, 1\}$ and the chosen action is a. Then the agent's payoff is $-(\omega - a)^2$ while the principal's payoff is $-(\omega + \beta - a)^2$ for some $\beta \in \mathbb{R}$.

   Find the values of the parameter β for which the principal delegates the decision to the agent.

2. **[30 points]** Let G be the following two-person normal-form game

	A	B
A	$4,4$	$3,6$
B	$6,3$	$0,0$

   Consider the repeated game $G(\infty, \delta)$ and the strategy profile (s_1, s_2) which be described as follows. There are two phases, 1 and 2. The initial history h^0 belongs to phase 1. In phase 1 the players play A. The play remains in phase 1 unless there is a unilateral deviation from (A, A). If there is a unilateral deviation, the play goes to phase 2. In phase 2 the players play (B, B). If there are no deviations or two deviations from (B, B) the play returns to phase 1 (i.e., phase 2 lasts one period). If there is a unilateral deviation from (B, B) then phase 2 restarts.

   Find the values of the discount factor δ for which (s_1, s_2) is a subgame perfect equilibrium of $G(\infty, \delta)$.

3. **[40 points]** An agent has to decide whether to implement a change or to keep the status quo. The optimal decision depends on the state of the world which is either g or b. The probability of the state g is $\frac{3}{10}$. The agent's payoff is equal to 1 if he implements

the change and the state is g and equal to zero if he implements the change and the state is b. The agent obtains a payoff of $1/2$ in both states if he keeps the status quo.

The agent does not observe the state. However, he can rely on the report of an expert who observes the state. There are two possible reports: γ and β. The expert *commits* to a reporting strategy $\left(\mu_g, \mu_b\right) \in [0,1]^2$ with $\mu_g \geq \mu_b$. When the state is $\omega \in \{g, b\}$, the expert sends the report γ with probability μ_ω and the report β with probability $1 - \mu_\omega$. In both states the expert's payoff is equal to 1 if the agent implements the change and equal to 0 if the agent keeps the status quo (notice that the expert's payoff does not depend on the state).

The timing of the game is as follows. First, the expert chooses the reporting strategy $\left(\mu_g, \mu_b\right)$. Then he observes the state and sends a report according to $\left(\mu_g, \mu_b\right)$. The agent observes the reporting strategy $\left(\mu_g, \mu_b\right)$ and the report (but not the actual state) and makes the final decision.

Construct a perfect Bayesian equilibrium.

(Hint: Can the expert induce the agent to implement the change after receiving both the report γ and the report β? Let $\Pr(g|\gamma)$ and $\Pr(g|\beta)$ denote the agent's belief that the state is g after receiving the report γ and β, respectively. It is easy to check that the expected value of $\Pr(g|\gamma)$ and $\Pr(g|\beta)$ is equal to the prior $\frac{3}{10}$.)