Communication and Voting with Double-Sided Information*

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Abstract

We analyze how communication and voting interact when there is uncertainty about players' preferences. We consider two players who vote on forming a partnership with uncertain rewards. It may or may not be worthwhile to team up. Both players want to make the right decision but differ in their attitudes toward making an error. Players' preferences are private information and each player is partially informed about the state of the world. Before voting, players can talk to each other.

We completely characterize the equilibria and show that communication is beneficial. The main role of communication is to provide a double check: When there is a conflict between a player's preferences and her private information about the state, she votes in accordance with her private information only if it is confirmed by the message she receives from her opponent. In a scenario where only one of the players is allowed to talk, the benefits of communication are independent of the identity of the sender.

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1 Introduction

Two people have to decide whether to form a partnership with uncertain rewards. In one state of the world it is worthwhile to team up, in another it is not. The players can make two mistakes: They can either form the partnership when it is not worthwhile to do so, or they can decide not to team up despite there being gains from the partnership. Both players want to make the right decision but they have different concerns about the two errors. A player's preferences are known to herself but not to her opponent. Each player is partially informed about the gains of the partnership because she receives a signal from nature that is correlated with the unknown state of the world. The players decide about forming the partnership by voting and the partnership is formed if both vote in its favor. Before voting, the two players can talk to each other. This gives them an opportunity to exchange their private information and make a more informed decision.

The essence of this scenario is the interaction of communication and voting in an environment in which players have different and potentially conflicting preferences. One might think, for example, of two executives who have just met and hence do not have full knowledge of each other's attitudes. The two executives are trying to decide whether or not to form an alliance. They must both agree in order for the alliance to be established. Given that the benefits from the alliance are uncertain and both executives know something about the state of the world, they have a natural incentive to talk to each other and pool the available information. Our goal is to analyze the functions and benefits of communication in situations like this.

Communication is especially important whenever a small group of people makes a joint decision. In general, voting by itself fully aggregates information in large elections (Feddersen and Pesendorfer 1997, Feddersen and Pesendorfer 1998, Duggan and Martinelli 2001, Gerardi 2000). This is not the case for small electorates. This means that a small committee may reach a decision different from the one that would be made if all information were publicly available (see also Austen-Smith and Banks (1996) and Li, Rosen and Suen (2001)).

However, whenever a small group of people needs to reach a decision, communication takes place before votes are cast. It is easy to see that communication may help the group make a better decision if all players have the same preferences. In this case each player has an incentive to truthfully reveal her private information and the group thus adopts the decision that would have been preferred by everyone were all information publicly available. In other words, information is fully aggregated once communication is added to voting. This argument extends to the case where preferences are sufficiently close (but not identical) and common knowledge (Austen-Smith 1990, Coughlan 2000).

It is an open question what happens when preferences differ. When players differ in their attitudes toward the two errors, each has an incentive to take advantage of the opportunity to communicate in order to manipulate the decision in the direction of her bias. On the other hand, players also have an incentive to reveal their private information truthfully in order to reach a more informed decision. These opposing incentives lead to more complex equilibrium behavior than in the case when preferences are similar and only the latter incentive matters.

This paper is the first to analyze the interaction of communication and voting when players have different and perhaps conflicting preferences. In contrast to the case of homogeneous preferences, our model allows us to gain a better understanding of the importance of communication in a voting game.

In our analysis, we assume that preferences are private information. This is characteristic of situations like partnership formation or jury deliberations in which players do not have a long history of interactions and hence do not know each other's attitudes with certainty. Our assumption of uncertain preferences also facilitates part of the analysis because it allows us to focus on symmetric equilibria.

We consider two related games, one in which only one player is allowed to talk and another one in which both players are allowed to do so. We provide a complete characterization of the equilibria in both games and show that, while not all private information is revealed, some information transmission takes place.

In the scenario in which only one player talks, we start by considering the case where the sender and the receiver have information of the same quality, i.e. the degree of correlation between the state of the world and their signal is the same. We show that the purpose of communication is to serve as a *double check*. When there is a conflict between the receiver's concern with the two errors and her private information about the state of the world, she votes in accordance with her private information only if it is confirmed by the message she receives from the sender. On the other hand, a sender in the same situation resolves this conflict by *delegating* the final decision to the receiver. She votes in favor of the alternative option and thus allows the final decision to depend solely on the receiver's vote. Next we allow the two players to have private information of different quality. In this case it is natural to investigate which player should assume the role of the sender. Surprisingly, the identity of the sender is *irrelevant* in the sense that it does not affect the quality of the final decision. In our model, information can be aggregated either in the communication stage or in the voting stage. Our irrelevance result suggests that communication and voting are "perfect substitutes": All the information that is not transmitted by the sender's message is aggregated by the players' votes.

Allowing both players to talk complicates the analysis because players are now simultaneously senders and receivers of communication. In order to make the analysis tractable, we assume that their information is of the same quality. Our analysis highlights the same functions of communication as in the one-sender game. In particular, communication provides players with a double-check when their private information conflicts with their relative concerns. To evaluate the benefits of communication, we compare players' utilities in the one- and two-sender games with their utilities in a pure voting game (Gerardi 2000). We find that welfare is increasing in the number of speakers, but also that there are decreasing returns to scale in a certain sense.

This paper contributes to a growing literature that studies the effects of communication in various environments such as auctions (Campbell 1998), bargaining (Farrell and Gibbons 1989, Matthews 1989), agenda setting (Ordeshook and Palfrey 1988), the provision of public goods (Palfrey and Rosenthal 1991), and entry in natural-monopoly industries (Farrell 1987). These papers show that communication dramatically affects the set of outcomes and that its welfare effects depend on the underlying game. Given the attention that voting has received in the recent literature, it is surprising that communication has not been analyzed in this context, especially since communication always takes place when a small committee has to reach a joint decision. Our analysis allows us to describe the important role that communication plays in collective decision-making processes.

The remainder of the paper is organized as follows. Section 2 presents the one-sender game. In Section 3 we analyze the two-sender game. Section 4 concludes and suggests a number of possible extensions. All proofs are relegated to the appendices.

2 One-Sender Game

2.1 Model Setup and Equilibrium Concept

Two players jointly decide whether to maintain the status quo or to change it by adopting some alternative option. Let the joint decision to maintain the status quo be denoted by $d = 1$ and the decision to change it by $d = 0$. The rewards from the decision depend on the unknown state of the world ω, which takes on the values 0 or 1 with equal probability.[1] A decision that matches the state of the world ($d = \omega$) is optimal for both players, and we call such a decision correct in what follows. There are two types of errors: adopting the alternative option ($d = 0$) when the state is $\omega = 1$ and maintaining the status quo ($d = 1$) when the state is $\omega = 0$.

Players differ in their relative concerns with the two errors. A player's preferences are formally captured by assigning her a type $q \in (0, 1)$. Types are private information, but it is common knowledge that each type is an i.i.d. draw from a distribution F with domain $(0, 1)$, where F is continuous, strictly increasing, and admits a density f such that $f(0^+) > 0$ and $f(1^-) > 0$. A player's utility $u(d, \omega, q)$ depends on the joint decision d, the state of the world ω, and her type q. We normalize the utility from making a correct decision to zero and set $u(0, 0, q) = u(1, 1, q) = 0$, $u(0, 1, q) = -q$, and $u(1, 0, q) = q - 1$. Hence, higher types are more concerned with adopting the alternative option when the correct decision is to maintain the status quo than lower types.

Player i is partially informed about the state of the world because she receives a signal $s_i \in \{0, 1\}$ from nature which is correlated with the state of the world: $\Pr(s_i = \omega|\omega) = p_i \in (1/2, 1)$. If the degree of correlation between the state of the world and their signal is the same, we say that the players have private information of the same quality. Otherwise, the quality of their private information differs. Conditional on the state of the world, the signals are independent across players.

The game proceeds in two stages. In the first stage, one of the players, the *sender*, expresses her opinion in the form of a straw vote. This means that the sender sends a message $m \in \{0, 1\}$ to her opponent, the *receiver*. In what follows we index variables pertaining to the sender and the receiver by s and r, respectively. In the second stage, player i casts her vote $v_i \in \{0, 1\}$. The vote $v_i = 1$ is in favor of the status quo and

[1] Our results hold also when $\Pr(\omega = 0) \neq \Pr(\omega = 1)$.

the vote $v_i = 0$ is in favor of the alternative option. Voting is simultaneous and the decision rule prescribes that the alternative is adopted whenever both players vote for it; otherwise the status quo is maintained. For clarity of exposition, we drop indices in the treatment when no ambiguity arises.

The sender's strategy consists of two choices. The message choice is described by a function assigning to each pair (q, s) the probability that a sender of type q sends message $m = 1$ after observing signal s. Her voting choice is described by a function assigning to each triplet (q, s, m) the probability that a sender of type q votes $v = 1$ after she has observed signal s and sent message m. The receiver's strategy is a function assigning to each triplet (q, s, m) the probability that a receiver of type q votes $v = 1$ when she observed signal s and received message m.

In order to characterize the equilibria of this game, we introduce so-called *cutoff strategies*. We say that the sender's message strategy has a cutoff structure if for any $s \in \{0, 1\}$, there exists a number q_s in the unit interval such that, after observing signal s, she reports message $m = 0$ ($m = 1$) if her type q is smaller (larger) than q_s. The sender's voting strategy has a cutoff structure if for any $s \in \{0, 1\}$ and $m \in \{0, 1\}$, there exists a number q_{sm} in the unit interval such that, after observing signal s and sending message m, she casts the vote $v = 0$ ($v = 1$) if her type q is smaller (larger) than q_{sm}. Likewise, the receiver's cutoff r_{sm} prescribes that, after observing signal s and receiving message m, the receiver votes $v = 0$ if $q < r_{sm}$ and $v = 1$ if $q > r_{sm}$. In other words, *ceteribus paribus* high types send message $m = 1$ and vote $v = 1$ whereas low types send message $m = 0$ and vote $v = 0$.

Our solution concept is Perfect Bayesian Equilibrium (PBE) with the additional requirement that players do not use weakly dominated strategies. In Appendix A.1, we show that every PBE is outcome-equivalent to a PBE in which players use cutoff strategies.[2] We henceforth restrict attention to PBE in cutoff strategies.

Not all cutoffs are necessarily identified by sequential rationality or weak dominance. Suppose that the sender's cutoff strategy is such that $q_{01} < q_0$ and $q_{11} < q_1$. This means that after sending message $m = 1$ the sender vetos change. Hence, if the receiver observes message $m = 1$, she knows that the sender will vote $v = 1$ and that the outcome will be $d = 1$ irrespective of her vote. It follows that the receiver's opti-

[2]Two strategy profiles are outcome-equivalent if they induce the same probability distribution over final decisions. Note that unlike the case of voting without communication, ruling out weakly dominated strategies does not guarantee that all equilibria admit a cutoff structure.

mal strategy is not uniquely determined. Similarly, a sender of type $q > \max\{q_{s0}, q_{s1}\}$ knows that she will vote $v = 1$ after observing signal s regardless of the message she sends. Even though her message may affect the receiver's vote, the final decision is $d = 1$. Since the sender is indifferent between messages, her optimal strategy is undetermined.

These examples suggest that an indeterminacy arises where an action does not have an impact on the final decision. In the case an action affects the outcome, the cutoffs are uniquely identified and related across signals by the family of functions

$$k_p(q) = \frac{q(1-p)^2}{q(1-p)^2 + (1-q)p^2}, \tag{1}$$

defined on $(0, 1)$ and indexed with the quality of the signal p. Specifically, $q_1 = k_{p_s}(q_0)$ and for $m \in \{0, 1\}$, $q_{1m} = k_{p_s}(q_{0m})$ and $r_{1m} = k_{p_r}(r_{0m})$ (see proof of Proposition 1). For future reference note that $k_p(q) < q$ for every q and p and that k_p is strictly increasing in q.

Hence, in order to resolve the indeterminacy mentioned above, we restrict attention to PBE in which *all* cutoffs are linked across signals through the functions k_{p_s} and k_{p_r}. This requirement may be justified by noting that these PBE are the only ones that survive a stability check: Consider a player and slightly perturb her opponent's strategy, so as to yield a unique best reply. We require that such a sequence of unique best replies converges to the original strategy as the perturbation vanishes. We call a PBE that satisfies this requirement *robust*. In Appendix A.2 we show that in a robust cutoff PBE *all* cutoffs are linked across signals through the functions k_{p_s} and k_{p_r}.

In order to conduct our analysis, we pose a technical assumption on the distribution of types. For any $q \in [0, 1]$, the ratio

$$\frac{F(x) - F(q)}{F(k_p(x)) - F(k_p(q))} \tag{2}$$

is strictly decreasing in x, $x \in [0, 1]$, $x \neq q$. This assumption is satisfied by all Beta distributions and should thus not be considered too restrictive.

2.2 Equilibrium Characterization

This game admits different equilibria. In so-called babbling and non-responsive equilibria communication plays no role: In a babbling equilibrium the sender's choice of message is independent of the signal she observes, in a non-responsive equilibrium the receiver's vote is independent of the message she receives. Non-responsive and babbling equilibria are outcome-equivalent (see Appendix A.1) and are also outcome-equivalent to the equilibria of the pure voting game analyzed by Gerardi (2000). When no communication takes place, the symmetric equilibrium is characterized by two cutoffs, $\tilde{q}_1$ and $\tilde{q}_0$. Accordingly, the voting behavior can be classified into three categories. Types with $q < \tilde{q}_1$ always vote $v = 0$ in favor of the alternative option and types with $q > \tilde{q}_0$ always vote $v = 1$ in favor of the status quo. Finally, types with $\tilde{q}_1 < q < \tilde{q}_0$ vote $v = s$ according to their signal. In summary, extreme types who are overly concerned with a particular error, vote according to their relative concerns whereas moderate types vote according to their signal.

Our goal is to analyze the interaction of communication and voting, so it is appropriate to consider *responsive* equilibria. In a responsive equilibrium, the receiver conditions her vote on her type, her signal, and the sender's message. The sender conditions her voting strategy not only on her type and her signal, but also on the message she sends. This is because she chooses a best reply to her opponent's equilibrium play which in turn changes according to the message she sends.

Next we characterize responsive equilibria in terms of configurations. By a configuration we mean the order of the cutoffs for a *given* signal. Although there is a large number of configurations, Proposition 1 shows that exactly one of them is consistent with a responsive PBE. This allows us to compute the responsive robust cutoff equilibria of the one-sender game by solving a system of equations (see the proof of Proposition 1).

Proposition 1 *Responsive robust cutoff equilibria exist and display the configuration*

$$0 < q_s < q_{s0} < q_{s1} < 1 \text{ for } s \in \{0, 1\},$$

$$0 < r_{s1} < r_{s0} < 1 \text{ for } s \in \{0, 1\}.$$

There does not exist a responsive robust cutoff equilibrium in any other configuration.

Fix a signal s. Proposition 1 shows that receivers with type $q \in (0, r_{s1})$ vote for the alternative option independent of the message they receive and receivers in $(r_{s0}, 1)$ always veto change. In contrast, receivers in (r_{s1}, r_{s0}) vote according to the message they receive ($v = m$). It follows that the receiver is (ex-ante) more likely to vote for the alternative option after receiving a message in favor of it than after observing a message against it. The main reason is that the sender's signal is *partially revealed* in equilibrium. In fact, in a robust PBE we have $q_1 = k_{p_s}(q_0) < q_0$. This implies that a sender with type $q \in (q_1, q_0)$ truthfully reports her signal at the message stage ($m = s$). Because some senders are truthful, receiving a particular message m strengthens the receiver's belief that the state of the world is indeed $\omega = m$. This in turn increases the probability that the receiver votes according to the message ($v = m$) because she would like the final decision to match the state of the world. Consequently, the sender can use her message to *manipulate* the receiver's vote.

The sender, in turn, has to choose among three behaviors. First, she can veto change ($v = 1$). Second, she can send a message in favor of the status quo but then vote for the alternative option ($m = 1$ and $v = 0$). Third, the sender can both express herself and vote in favor of change ($m = 0$ and $v = 0$). The first course of action guarantees that the status quo persists ($d = 1$) independent of the sender's message and the receiver's strategy. In both the second and the third course of action, the sender in effect concedes the final decision to the receiver. On the other hand, given the sender's ability to manipulate the receiver's vote, the second strategy makes the outcome $d = 1$ more likely than the third one.

Proposition 1 shows that the sender's behavior is very intuitive. High types with $q > q_{s1}$, who are very concerned with incorrectly adopting the alternative option, veto it.[3] Low types with $q < q_s$ are very concerned with mistakenly maintaining the status quo. They therefore maximize the probability that the final decision is to change the status quo by targeting their actions at achieving change ($m = 0$ and $v = 0$). Lastly, types with $q \in (q_s, q_{s1})$ are not overly concerned with a particular mistake and adopt an intermediate behavior by sending a message in favor of the status quo but then voting against it ($m = 1$ and $v = 0$).

A sender with type $q \in (q_{s0}, q_{s1})$ exhibits a rather counterintuitive feature: She reports a message in favor of the status quo ($m = 1$) in order to increase the probability that the alternative option is adopted ($d = 0$). Since $q \geq q_{s0}$, if she had sent message

[3] While these types are indifferent between the two messages, a responsive PBE exists only if they report send a message in favor of the status quo.

$m = 0$, she would then vote $v = 1$ and the status quo would be maintained. On the other hand, since $q \leq q_{s1}$, sending message $m = 1$ implies that she then votes $v = 0$ which, in turn, ensures that the alternative is adopted with positive probability.

Proposition 1 identifies the order of the cutoffs for a given signal. To gain additional insights into the role of communication, we now relate players' cutoffs across signals. The exact values of the cutoffs q_{00} and q_{10} are irrelevant. Recall that the threshold q_{s0} governs the voting behavior of senders who report message $m = 0$. However, provided that $q_{s0} > q_s$, the voting behavior of types who send message $m = 0$ does not depend on the specific value of q_{s0}. In fact, all types who send a message in favor of change subsequently vote for it.

We have already shown in Proposition 1 that $q_s < q_{s1}$ and $r_{s1} < r_{s0}$ and that cutoffs are related across signals by the functions k_{p_s} and k_{p_r} in a robust responsive cutoff equilibrium. Since $k_p(q) < q$, we conclude that q_1 (q_{01}) is the smallest (largest) cutoff for the sender. Similarly, r_{11} (r_{00}) is the smallest (largest) cutoff for the receiver. Therefore, it remains to determine the relationship between q_{11} and q_0 as well as r_{10} and r_{01}. Corollary 1 gives the order of these thresholds when the quality of the signal is the same for both players. In Section 2.3, we explore the case of different qualities.

Corollary 1 *If $p_s = p_r$, then $q_{11} < 1/2 < q_0$ and $r_{10} < 1/2 < r_{01}$.*[4]

Proposition 1 and Corollary 1 together imply the following orders of the relevant cutoffs when the two players have information of the same quality:

$$0 < q_1 < q_{11} < q_0 < q_{01} < 1,$$

$$0 < r_{11} < r_{10} < r_{01} < r_{00} < 1.$$

Figure 1 illustrates this result. The upper part of Figure 1 summarizes the equilibrium behavior for different types of senders. For each interval of types, the left (right) column reports the equilibrium strategy of the sender when she observes signal $s = 0$ ($s = 1$). The top row indicates the sender's message, the bottom row her vote. In the lower part of Figure 1, we describe the receiver's equilibrium behavior. For each interval of types, the left (right) column refers to signal $s = 0$ ($s = 1$). The top (bottom) row reports her vote after receiving message $m = 0$ ($m = 1$). To facilitate the

[4] In general these cutoffs are separated by $\left(1 + \frac{\Pr(\omega=1)}{\Pr(\omega=0)}\right)^{-1}$.

discussion, we give names to the senders and receivers in the five intervals. From left to right, we label them as *left extremist*, *left sophisticated*, *central*, *right sophisticated*, and *right extremist* types. We label some types sophisticated because their behavior is fairly complex, but that does not imply that the extremist and central types are boundedly rational. The label extremist reflects the fact that these types adopt a rather inflexible behavior.

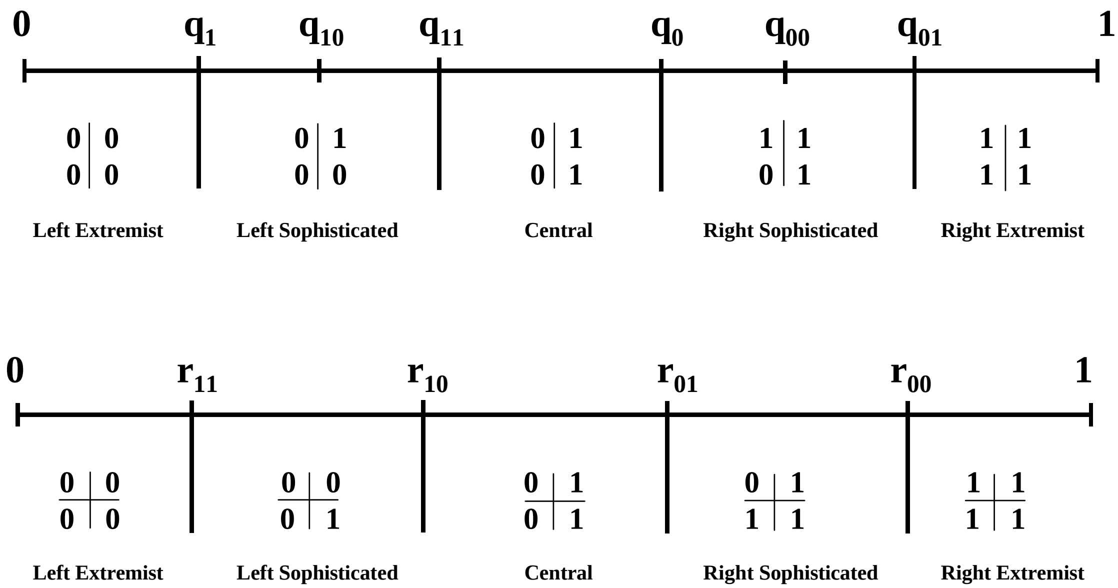

Figure 1: Path of play in the one-sender game. Same quality of information.

Consider the receiver first. Left extremist receivers are so concerned with the possibility of foregoing a valuable alternative that they vote against the status quo ($v = 0$) regardless of their signal and the sender's message. Right extremist receivers, conversely, always enforce the status quo ($v = 1$). Similar to extremists, central receivers never listen to the sender's message. However, unlike extremists, they vote according to their signal ($v = s$). Central types are not overly concerned with either type of error, so they have an incentive to use the available information. However, they know that the sender may misreport her signal, and thus prefer to vote according to their own signal.

Sophisticated receivers are the only types who may listen to the sender's message. Left sophisticated receivers, for example, are more concerned with mistakenly foregoing a good opportunity for change than with the opposite mistake (but less so than left extremists). These receivers therefore require more evidence in order to vote for the status quo than against it. Observing signal $s = 0$ is sufficient evidence for them that the final decision should be to adopt the alternative option. Therefore, left sophisticated receivers disregard the sender's message and vote in favor of change in this situation. Observing signal $s = 1$ is not conclusive, so in this case left sophisticated receivers require that their signal is confirmed by their sender's message in order to vote for the status quo. Hence, left sophisticated senders vote $v = 1$ if and only if their private information is in favor of change ($s = 1$) and that indication is confirmed by a message that is favorable to change ($m = 1$). Conversely, right sophisticated receivers vote in favor of change ($v = 0$) if and only if their private information and the sender's message support this decision ($s = 0$ and $m = 0$). In short, a sophisticated receiver uses the sender's message as a *double check*. Whenever her private information about the state of the world conflicts with her preferences, a sophisticated receiver will use the sender's message to check the validity of her own signal and will vote according to it only if it matches the sender's message. As far as a player's type may be interpreted as capturing her attitudes with respect to the final decision, we could say that when players have similar quality of information the main role of communication is to resolve a conflict between a player's knowledge and her ex-ante view of the world.

Consider the sender next. A left extremist is especially concerned with maintaining the status quo ($d = 1$) when the correct decision is to adopt the alternative option ($\omega = 0$). She thus votes for the alternative option ($v = 0$) and also sends message $m = 0$ in order to manipulate the receiver to do the same. Right extremists are overly concerned with adopting the alternative option ($d = 0$) when the status quo should be maintained ($\omega = 1$) and therefore veto change. Central senders condition their voting and message behavior on their signal ($m = v = s$). Similar to central receivers, central senders are not overly concerned with a particular mistake and thus use their private information.

A signal in favor of change ($s = 0$) persuades left sophisticated senders that the status quo should be abandoned. In this case, they express themselves in favor of the alternative option both at the message and at the voting stage ($m = 0$ and $v = 0$). However, after observing signal $s = 1$, left sophisticated senders adopt an intermediate behavior. If they were to veto change, that would determine the final outcome, so they vote $v = 0$ just as left extremists do. However, they are not as concerned as the left extremists about mistakenly maintaining the status quo, so they report message

$m = 1$ and *delegate* the final decision to the receiver. Right sophisticated senders display analogous behavior. They adopt the intermediate strategy of sending message $m = 1$ and delegating the final decision ($v = 0$) after observing signal $s = 0$. After observing signal $s = 1$, they are persuaded that the status quo should be maintained and determine the final outcome by vetoing change ($v = 1$).

Our result that a sophisticated sender delegates the final decision to the receiver parallels the literature on delegation. Li and Suen (2001) consider a simpler model with an uninformed principal and an informed agent. They show that extremist principals dictate the final decision whereas more moderate principals delegate it to the agent.

2.3 Quality of Information

The previous section focused on the case where the quality of information is the same for both players. If the signal qualities are different ($p_s \neq p_r$), then it is no longer true that all cutoffs pertaining to $s = 1$ are smaller than the cutoffs for $s = 0$. This may give rise to a variety of behaviors.

From Proposition 1 we know that, independent of the signal qualities, the smallest (largest) cutoff of the sender is q_1 (q_{01}) and that the cutoffs q_{10} and q_{00} do not affect the equilibrium behavior on path. Analogously, r_{11} (r_{00}) is the smallest (largest) cutoff of the receiver. It follows that the quality of information can affect only the relationship between q_{11} and q_0 as well as r_{10} and r_{01}. Hence, only the behavior of central senders and central receivers can depend on the quality of information.[5] Specifically, if $q_{11} > q_0$, central senders send $m = 1$ and vote $v = 0$ independent of their signal. If $r_{10} > r_{01}$, central receivers disregard their signal and vote $v = m$ according to the sender's message. Figure 2 illustrates the different combinations of equilibrium behavior for a uniform distribution of types. These combinations are indicated by circles, pluses, and squares.

If the quality of information is almost the same for both players, $p_s \approx p_r$, the order of the cutoffs is $q_{11} < q_0$ and $r_{10} < r_{01}$ (pluses in Figure 2). The resulting behavior is the same as discussed in Section 2.2. In particular, central senders and receivers behave according to their own signal. Consider for example a central sender. She has an incentive to use all available information which consists of her own signal

[5] We continue to label types from left to right.

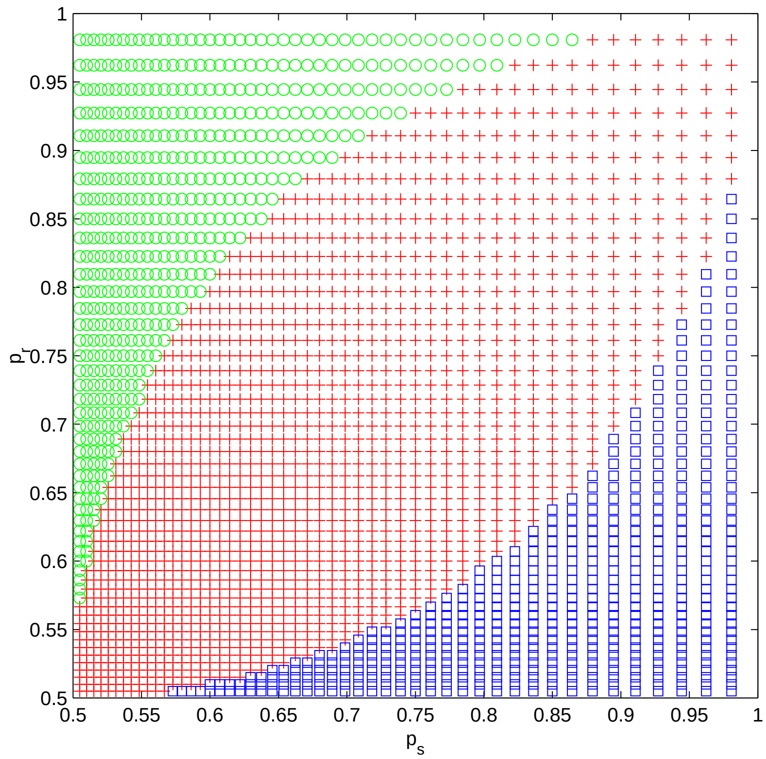

Figure 2: Equilibrium behavior as a function of the quality of information.

and the information contained in the receiver's vote. While observing signal $s = 1$ increases the conditional probability of state $\omega = 1$, the information contained in the receiver's vote either increases it ($v_r = 1$) or decreases it ($v_r = 0$). In order to evaluate the information content of the receiver's vote, the sender has to take into account that not all types of receivers vote truthfully. Therefore, the sender "discounts" the information content of the receiver's vote accordingly. If the quality of the signal is similar for both players, then the sender's updated probability that the state is $\omega = 1$ is greater than $1/2$ after she has observed signal $s = 1$ even if her own signal conflicts with her opponent's vote.

If the quality of the sender's information is much lower than the quality of the receiver's information, $p_s \ll p_r$, then $q_{11} > q_0$ and $r_{10} < r_{01}$ (circles in Figure 2). In this case, a poorly-informed central sender disregards her own signal and delegates the decision to the well-informed receiver. To continue the above example, the sender knows that the discounted information content of her opponent's is higher than the information content of her own signal. This is enough evidence to convince her that the final decision should coincide with her opponent's vote.

If the quality of the sender's information is much higher than the quality of the receiver's information, $p_s \gg p_r$, on the other hand, we obtain $q_{11} < q_0$ and $r_{10} > r_{01}$ (squares). When the sender is much better informed than the receiver, central receivers disregard their own signal and listen to the sender's message instead.

We next consider extreme signal qualities. One might think that if the sender has a perfect signal, then she should reveal her signal truthfully and all types of receivers should vote according to the sender's message. While this in fact constitutes a responsive PBE in the limiting case of $p_s = 1$, there are other equilibria. Our numerical analysis highlights one of them. As the probability that the sender receives the correct signal approaches unity, all senders are central and choose $m = v = s$ whereas the receivers in the interval $(0, 1/2)$ are left extremists and the ones in $(1/2, 1)$ are central (and thus vote $v = m$ for every s). Hence, the correct decision is reached in two different ways. A left extremist receiver always votes $v = 0$ and thereby delegates the final decision to the sender. On the other hand, a central receiver listens to the sender's message and votes $v = m$.

If the receiver has a very good signal, $p_r \approx 1$, all receivers are central and choose $v = s$ whereas the senders in the interval $(0, 1/2)$ are left extremists and the ones in $(1/2, 1)$ are central (and thus send message $m = 1$ and vote $v = 0$ for every signal s). The sender intends to use the information available to the receiver and delegates the decision to the receiver by voting $v = 0$. As the receiver disregards her message, the sender is basically indifferent between the two messages. However, a responsive PBE requires that left extremists choose $m = 0$ and central types $m = 1$.

2.4 Welfare Analysis

So far the distinction between the sender and the receiver has been exogenous. However, as the two players may have private information of different quality, a natural question arises concerning the identity of the sender. Who should send the message at the first stage? To answer this question we consider the ex-ante utilities of the players and the probabilities of the two possible wrong decisions (adopting $d = 1$ when $\omega = 0$ and choosing $d = 0$ when $\omega = 1$). In Proposition 2 we show that the identity of the sender is *irrelevant*. In particular, each player has the same ex-ante utility independent of whether she is the sender or the receiver. Similarly, the probabilities of the two errors when the better informed player sends a message coincide with the probabilities of the two errors when the sender is the player with the lower quality signal.[6]

Let $\Gamma(p, p')$ denote the game in which the sender gets the correct signal with probability p and the receiver observes the right signal with probability p'. We need to compare sets of equilibrium utilities because we have not established uniqueness of the equilibrium. Denote by $U_s(p, p')$ the set of the sender's ex-ante responsive cutoff equilibrium utilities in the game $\Gamma(p, p')$. Similarly, let $U_r(p, p')$ denote the set of the receiver's ex-ante utilities.

Proposition 2 *For any pair* $(p, p') \in \left(\frac{1}{2}, 1\right)^2$, $U_s(p, p') = U_r(p', p)$.

Proposition 2 shows that, from an ex-ante point of view, the two players are indifferent between being the sender or the receiver. This result could be also obtained by demonstrating that the set of equilibria of the game $\Gamma(p, p')$ is outcome-equivalent to the set of equilibria of the game $\Gamma(p', p)$. Proposition 2 would then follow as a straightforward corollary.

Underlying Proposition 2 is the fact that every responsive PBE (q, r) of the game $\Gamma(p, p')$ is related to a responsive PBE (q', r') of the game $\Gamma(p', p)$ by

$$q'_s = r_{s1}, \quad q'_{s1} = r_{s0}, \quad r'_{s1} = q_s, \quad r'_{s0} = q_{s1} \quad \text{for } s = 0, 1.$$

The first equality, for example, follows from the fact that the receiver in the game $\Gamma(p, p')$, after observing signal s and receiving message $m = 1$, conditions her voting decision on an event that has the same probability as the event on which the sender in the game $\Gamma(p', p)$ conditions her message decision after observing signal s. Using this symmetry result to compute $U_s(p, p')$ and $U_r(p', p)$ yields the relationship presented in Proposition 2.

Information can be aggregated either in the communication or in the voting stage. Proposition 2 suggests that communication and voting are "perfect substitutes" in the sense that all the information that is not transmitted by the sender's message is aggregated by the players' votes. This leads to the question of whether or not communication is beneficial at all. We defer an answer to this question to Section

[6] Our irrelevance result can be contrasted to Dekel and Piccione (2000) who consider a sequential voting game without communication. When players have identical preferences, they show that it is optimal to have the better-informed players vote earlier.

3.4, where we compare a voting game without communication to the one- and two-sender games.

3 Two-Sender Game

3.1 Model Setup and Equilibrium Concept

We now consider the case where both players participate in the straw vote. To keep the analysis tractable, we assume that the quality of information is the same for both players, i.e. $p_1 = p_2 = p$. Player i sends a message $m_i \in \{0, 1\}$. The outcome of the straw vote is common knowledge before the actual vote takes place. Hence, player i's voting strategy is a function assigning to each quadruplet (q, s, m, M) the probability that player i of type q votes $v = 1$ after she has observed signal s, sent message m, and received message M. Our solution concept is symmetric PBE. As in Section 2, we rule out weakly dominated strategies.

It is easy to show that for every PBE there exists an outcome-equivalent PBE in which the voting strategy admits a cutoff representation. Even though we are not able to establish a similar result for the message strategy, we follow the spirit of the one-sender case and focus on symmetric cutoff PBE. Such profiles are identified by the cutoffs q_s and q_{smM} with $s \in \{0, 1\}$, $m \in \{0, 1\}$, and $M \in \{0, 1\}$. This means that a player of type q sends message $m = 1$ ($m = 0$) after observing signal s if $q > q_s$ ($q < q_s$), and that a player of type q votes $v = 1$ ($v = 0$) after observing signal s, sending message m, and receiving message M if $q > q_{smM}$ ($q < q_{smM}$).

As in the one-sender model, some cutoffs are not uniquely identified by sequential rationality or weak dominance. We therefore restrict attention to robust equilibria in which all cutoffs are related across signals through the function k_p defined in equation (1) (see Appendix A.2 for details).

3.2 Equilibrium Characterization

The two-sender game admits babbling and non-responsive equilibria. It is easy to show that symmetric non-responsive and symmetric babbling equilibria are outcome-equivalent.[7] With a slight abuse of terminology, we henceforth call any equilibrium

[7] The proof is available upon request.

that is outcome-equivalent to a babbling equilibrium non-responsive. This includes some responsive equilibria that are also outcome-equivalent to babbling equilibria (see the proof of Proposition 3). This is because the final decision is the same although the voting strategy may be different.

We consider responsive equilibria in order to shed light on the interaction of communication and voting. In a responsive equilibrium, there are types of players who condition their vote on their opponent's message. In Proposition 3 we present a complete characterization of the responsive equilibria of the two-sender game.

Proposition 3 *There exist three classes of responsive robust cutoff equilibria:*

Class 0**:** $q_s < q_{s01} < q_{s11} < q_{s00} = q_{s10}$ *for* $s \in \{0,1\}$*;*

Class 1**:** $q_{s01} < q_s < q_{s00} < q_{s10}$ *and* $q_{s11} \leq q_s$ *for* $s \in \{0,1\}$*;*

Class 2**:** $q_{s11} \leq q_{s01} = q_s < q_{s00} = q_{s10}$ *for* $s \in \{0,1\}$*.*

There does not exist a responsive robust cutoff equilibrium in any other configuration.

For any class, the smallest cutoff in $s = 0$ *is larger than the largest cutoff in* $s = 1$ *and the two sets of cutoffs are separated by* $1/2$*. Moreover, the equilibria of class* 0 *are outcome-equivalent to the equilibria of class* 1*.*

Proposition 3 shows that there are three classes of equilibria and that two of them are outcome-equivalent. We defer a detailed discussion of the equilibrium behavior of classes 0 and 1 and next show that equilibria in class 2 are in turn outcome-equivalent to the equilibria of the one-sender model. Since we do not establish uniqueness, we need to compare sets of equilibria. To facilitate this comparison, we restrict attention to equilibria of the one-sender game in which $r_{s0} = q_{s1}$ and $r_{s1} = q_s$. Denote by $E(p)$ the set of outcomes induced by such robust responsive cutoff equilibria of the one-sender game. Similarly, let $E^2(p)$ denote the set of outcomes induced by equilibria of class 2.

Proposition 4 *For any* $p \in \left(\frac{1}{2}, 1\right)$*,* $E(p) = E^2(p)$*.*

In other words, for every equilibrium of class 2, the one-sender game admits an outcome-equivalent equilibrium. Conversely, each equilibrium of the one-sender game has an outcome-equivalent counterpart in class 2. This is unexpected because the equilibria of class 2 are symmetric and thus conceptually different from the equilibria of the one-sender game. In fact, while in the latter only one player communicates any information about the state of the world, in the equilibria of class 2, both players reveal some information. Note that it is not the case that in the equilibria of class 2 one sender babbles. Obviously, an *asymmetric* equilibrium of the two-sender game can be constructed as follows. Given a responsive equilibrium of the one-sender game, one player babbles at the message stage and chooses the same voting strategy as the receiver. The other player behaves as if she were the sender.

The on-path behavior in the equilibrium of class 2 is similar to the behavior in a robust responsive cutoff equilibrium of the one-sender game. In particular, all types in $(0, q_1)$ behave as left extremist senders, all types $q \in (q_{100}, q_0)$ mimic central senders, and all types in the interval $(q_{000}, 1)$ act as right extremist senders. Types in the interval (q_1, q_{100}) send message $m = 0$ and vote $v = 0$ upon observing $s = 0$. When the realized signal is $s = 1$, they send message $m = 1$ and then condition their vote on the opponent's message, voting $v = 0$ if and only if they receive message $M = 0$. Their voting behavior coincides with the voting behavior of the left sophisticated receivers in the one-sender game. At the same time, their message strategy is the same as the message strategy of the left sophisticated senders. One could say that these types combine the role of both sender and receiver. Analogously, the types in the interval (q_0, q_{000}) play the role of right sophisticated senders and receivers.

We now turn to the equilibria of class 0 and class 1. Figure 3 illustrates the equilibrium behavior. The top part of Figure 3 summarizes the different paths of play in equilibria of class 0, the bottom part refers to equilibria of class 1. For each interval, the first column reports the path of play of the type when she has received the signal $s = 0$ from nature, and the second column, the path after the signal $s = 1$. The first row identifies the message sent. The second row refers to the vote. When a type conditions her vote on her opponent's message, we first present the vote after receiving message $M = 0$ and second the vote after $M = 1$.

Since class 0 and class 1 are outcome-equivalent, we start by discussing the behavior implied by equilibria of class 1 and then comment on the differences between class 1 and class 0. To facilitate the discussion, we label the types from left to right as *left extremist*, *left non-revealing sophisticated*, *left truthful sophisticated*, *central*, *right truthful sophisticated*, *right non-revealing sophisticated*, and *right extremist* types. We now distinguish between sophisticated types who truthfully report their signal and

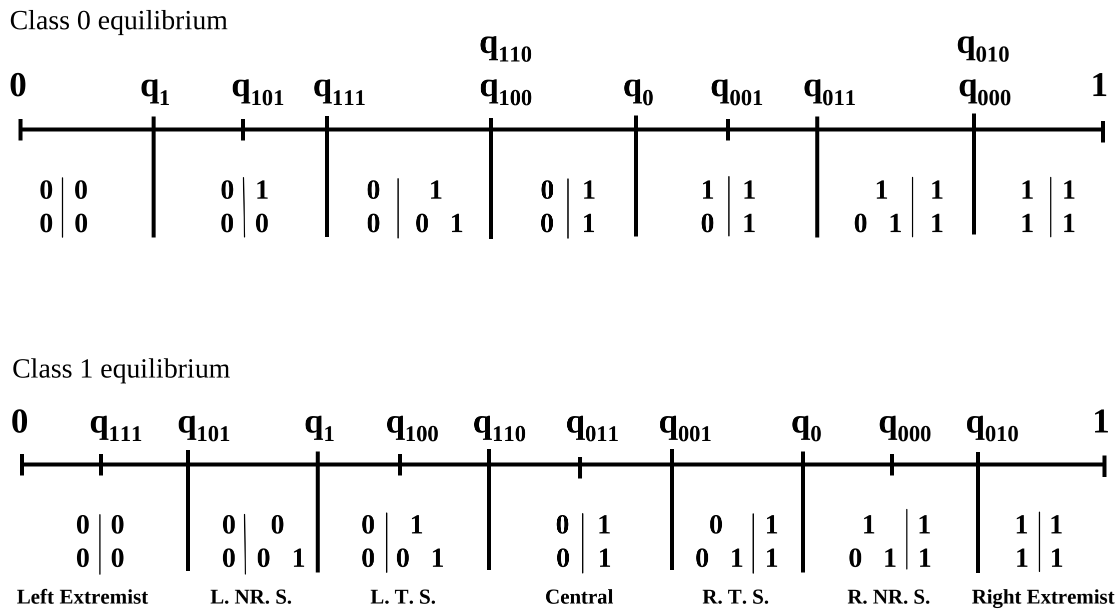

Figure 3: Path of play of class 0 and class 1 equilibria in the two-sender game.

those who do not condition their message on their signal.

As in the one-sender game, players *partially reveal* their private information. Specifically, left truthful sophisticated, central, and right truthful sophisticated types report the signal that they observe whereas the remaining types send a message that is independent of their signal. Turning to the voting behavior, a player is more likely to veto change after receiving a message in favor of the status quo. Formally, $q_{sm0} > q_{sm1}$ for all $s \in \{0, 1\}$ and $m \in \{0, 1\}$. This allows a player to *manipulate* her opponent.

To gain insight into a player's behavior in a PBE, let us fix her opponent's strategy and her signal. The player can decide the final outcome by voting $v = 1$. Alternatively, she can first send a message $m \in \{0, 1\}$ and then vote for change irrespective of her opponent's message ($v = 0$) or vote in line with her opponent's message ($v = M$).[8]

Suppose first that the player observes signal $s = 1$. Left extremists are very concerned with mistakenly maintaining the status quo. Therefore they send message $m = 0$ in order to increase the probability that their opponent votes in favor of the alternative option and vote $v = 0$ themselves. Left non-revealing sophisticated types continue to manipulate their opponent into voting for change by falsely sending message $m = 0$ but then listen to their opponent's message and vote $v = M$. Left

[8]Since there is a positive probability that her opponent reveals her signal truthfully, it is never optimal for the player to vote against her opponent's message.

non-revealing sophisticated types take into account that their opponent may vote for change because they have sent a message in favor of it. Since they are less concerned with mistakenly maintaining the status quo than left extremists, they follow their opponent's message at the voting stage in order to avoid that the status quo is maintained simply because of their own message. After receiving signal $s = 1$, left truthful sophisticated types are not overly concerned with a particular error and hence have no incentive to manipulate their opponent. In fact, these types report their signal truthfully and listen to their opponents message ($v = M$). Finally, central, right truthful sophisticated, right non-revealing sophisticated, and right extremist types are very concerned with erroneously adopting the alternative option and therefore veto change (and send message $m = 1$). To summarize, low types target their actions towards the alternative option whereas high types tend to favor the status quo.

Suppose next that the player observes signal $s = 0$. In a responsive PBE of class 1, we observe four different behavioral patterns. Specifically, left extremist, left non-revealing sophisticated, left truthful sophisticated, and central types disregard their opponent's message and express themselves in favor of change at the message and the voting stage ($m = 0$ and $v = 0$). Right truthful sophisticated types send message $m = 0$ and right non-revealing sophisticated types $m = 1$. However, both their votes reflect their opponent's message ($v = M$). Finally, right extremist types ensure that the status quo is maintained (and send message $m = 1$).

Figure 3 shows that in a responsive PBE of class 1, only truthful and non-revealing sophisticated types (left and right) make use of their opponent's message. In particular, these types use their opponent's message as a *double check* when their private signal conflicts with their concerns, similar to sophisticated receivers in the one-sender game. They vote according to their own signal only if it is confirmed by their opponent's message.

The behavior in equilibria of class 0 is the same as in equilibria of class 1 with two exceptions. In an equilibrium of class 0, left non-revealing sophisticated types reveal their signal and vote for change independent of their signal on their opponent's message. In addition, right truthful sophisticated types always send message $m = 1$ and then vote $v = s$ according to their own signal.

3.3 Quality of Information

We conduct a numerical analysis with a uniform distribution of types over a grid of values for the quality of information p. In each class, the numerical analysis led to a unique equilibrium.

For brevity we restrict the discussion to equilibria of class 1. In Figure 4 we present the shares of behavioral patterns for each level of quality of information. The size of left and right extremist types decreases in p (it is close to 1 when p approaches $1/2$ and it is about 0 for p near 1). The truthful sophisticated types grow steadily to pervade the entire interval $(0, 1)$ for p close to 1. The central types disappear for p close to $1/2$ and p close to 1. The mass of left and right non-revealing sophisticated is relatively small.

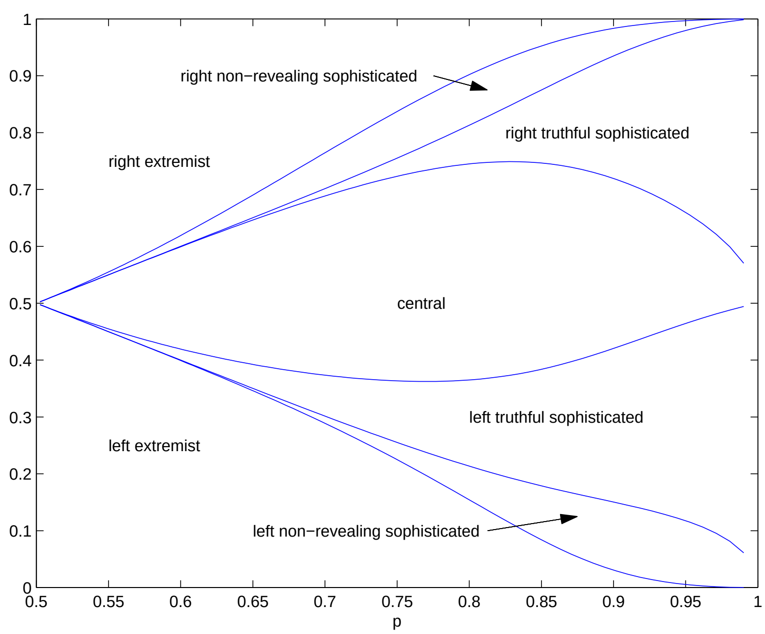

Figure 4: Shares of behavioral patterns in the class 1 equilibrium as a function of the quality of information.

When the quality of information is very poor, almost all players condition their behavior only on their relative concern with respect to the two possible mistakes and disregard both their own signal and their opponent's message. This is an intuitive results, because when private signals are not informative, the whole purpose of communication and voting to aggregate private information is defied.

If the quality of information is very good, almost all players send their message sincerely. Because of that, a player's message contains valuable information. On the other hand, since the player's own signal is very informative, a conflict between preferences and signal may arise even for extreme types. Therefore, almost all types are willing to condition their voting behavior on their opponent's opinion. In other words, they prefer to take advantage of the opponent's private information as a double check.

Finally, when the quality of the information is intermediate, there is partial revelation of information. Due to the presence of extremist types, a player has to "discount" the information content of her opponent's message. This explains the presence of central types who are not overly concerned with a particular mistake and thus disregard their opponent's message and vote according to their own signal.

3.4 Welfare Analysis

Intuitively, giving players the opportunity to talk cannot increase players' welfare when the signal is either completely uninformative or perfectly informative. However, communication reveals some additional information in the case when the quality of the signal is intermediate. This suggests that communication can increase players' welfare in this case.

We compare players' ex-ante equilibrium utility in the pure voting game, the one-sender game, and the two-sender game when the distribution of types is uniform. The result is illustrated in Figure 5, where the notation u_n refers to the ex-ante utility associated with the Pareto-dominant equilibrium of the n-sender game. The class 1 (and thus the class 0) equilibrium Pareto-dominates the equilibrium of class 2 which in turn is outcome-equivalent to the responsive PBE of the one-sender game. Moreover, all three classes of equilibria Pareto-dominate the equilibrium of the pure voting game. Our result that the ex-ante utility of all players is increasing in the number of speakers is in contrast to the literature on cheap talk originated by Crawford and Sobel (1982) where the principal's utility, but not necessarily an agent's utility, is increasing in the number of agents (Battaglini 2002, Krishna and Morgan 2001).

The difference in utility between the Pareto-dominant equilibrium in the two-sender game (class 1 and 0) and the equilibrium in the one-sender game is smaller than the difference between the equilibrium in the one-sender game and the equilibrium in the pure voting game. Since the increment of utility contributed by the first speaker

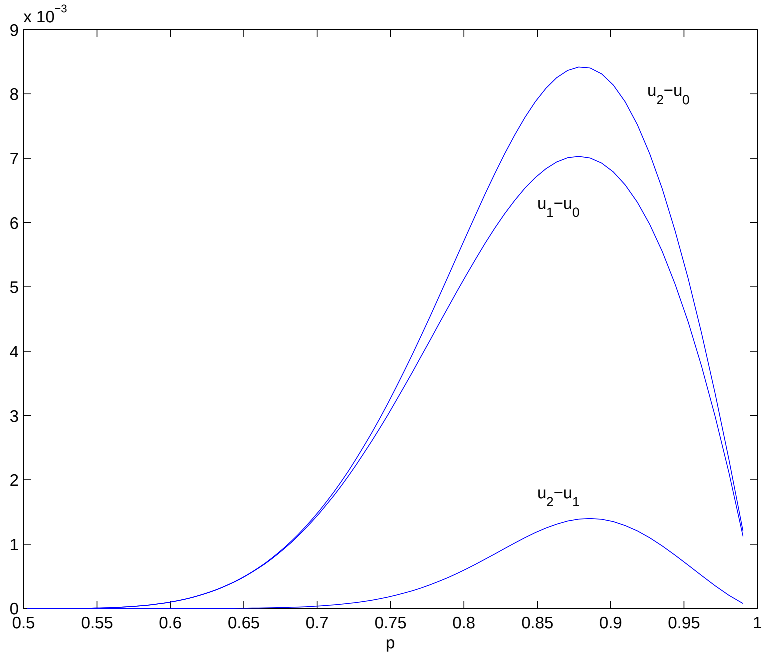

Figure 5: Utility comparison between the pure-voting game and the one- and two-sender games.

is larger than the increment of utility contributed by the second speaker, we could say that there is a case of decreasing returns to scale.

4 Conclusion

This paper analyzes the interaction of communication and voting in the context of a small committee (two players). When players share similar preferences, communication takes a very simple form since all players have an incentive to reveal their private information (Austen-Smith 1990, Coughlan 2000). However, in many situations players do not have a long history of interactions and hence do not have full knowledge of each others attitudes. Therefore, we consider the case where preferences are different and uncertain.

We provide a complete characterization of the equilibria both for the case where only one player is allowed to talk and for the case where both are allowed to do so. We show that, while not all private information is revealed, some information transmission takes place. Our main contribution is that we demonstrate how this information transmission benefits the players and helps them reach a better decision.

We show that when the two players have private information of the same quality about the state of the world, the purpose of communication is to serve as a double check. A player uses her opponent's message to resolve a conflict between her preferences and her private information. In the one-sender game the sender resolves this conflict by delegating the final decision to the receiver.

When the two players have private information of different quality and only of them is allowed to talk, we show that the identity of the sender does not affect the quality of the final decision. This suggests that communication and voting are "perfect substitutes" in the sense that all the information that is not transmitted by the sender's message is aggregated by the players' votes.

To evaluate the impact of communication on players' welfare, we compare the voting game without communication to the one- and two-sender games. We show that communication is beneficial and subject to decreasing returns to scale.

Our results imply one round of communication is not sufficient for all the available information to be transmitted. In general, allowing for more than one round of communication expands the set of possible outcomes (see for example Forges (1990) and Aumann and Hart (1999)). It would be interesting to explore the question whether allowing more complex forms of communication helps players to reach a more informed decision.

Adding communication to a voting game considerably complicates the analysis. We have therefore restricted attention to a setup with two players, two states of the world, and two signals. Future research should consider more general environments. For example, having more than two players would allow us to analyze and compare different voting rules. In sum, this paper is a first step toward understanding the role of communication in a collective decision-making process.

Appendix A: Cutoff Equivalence and Robustness

A.1 Cutoff Equivalence

In order to show that any PBE in undominated strategies is outcome-equivalent to some cutoff PBE, we first need to formally define players' strategies. The sender's behavioral strategy consists of two choices. The message choice is described by a measurable function $\mu : (0,1) \times \{0,1\} \to [0,1]$ where $\mu(q,s)$ denotes the probability that the sender sends message $m = 1$ when her type is q and she observes signal s. The voting choice is defined by a measurable function $\sigma : (0,1) \times \{0,1\}^2 \to [0,1]$ where $\sigma(q,s,m)$ denotes the probability that the sender votes $v = 1$ when her type is q, she has observed signal s, and she has sent message m. The receiver's strategy is described by a measurable function $\rho : (0,1) \times \{0,1\}^2 \to [0,1]$ where $\rho(q,s,m)$ denotes the probability that the receiver votes $v = 1$ when her type is q, she has observed signal s, and she has received message m.

Associated with each strategy profile is a distribution function which assigns to each quadruplet (q^s, q^r, s_s, s_r) the probability that the decision $d = 1$ is made by a sender of type q^s and a receiver of type q^r, when they observe signals s_s and s_r, respectively. Two strategy profiles are outcome-equivalent if their associated distribution functions are equal almost everywhere.

Proposition 5 *Every PBE in undominated strategies is outcome-equivalent to a PBE in which the sender's message and voting strategies admit a cutoff structure described by (q_s, q_{sm}), and the receiver's strategy admits a cutoff structure described by r_{sm}.*

Proof. In the proof we distinguish between responsive and non-responsive PBE. An equilibrium is non-responsive if $\rho(q,s,0) = \rho(q,s,1)$ for $s \in \{0,1\}$ and for any q. A babbling equilibrium satisfies $\mu(q,0) = \mu(q,1)$ for any q. Babbling equilibria are non-responsive, but not necessarily vice versa.[9] Moreover, any babbling PBE is outcome-equivalent to a cutoff PBE where $q_0 = q_1 = 0$. So we need to show that non-responsive and babbling equilibria are outcome-equivalent.

Lemma 1 *Non-responsive and babbling equilibria are outcome-equivalent.*

This result follows from the fact that if the receiver's voting behavior is independent of the message, then also the sender's voting choice (and thus the final decision) is independent of the message. The formal derivation is available upon request.

Turning to responsive PBE, we proceed in three steps. First we spell out the equations that characterize the equilibrium, then we show that the set of types who

[9]Note that in this game there are non-responsive equilibria which are not babbling. For instance, $\mu(q,0) = \mu(q,1) \in (0,1)$ for almost every q, $\sigma(q,s,0) = \sigma(q,s,1)$ and $\rho(q,s,0) = \rho(q,s,1)$ (where $\sigma(q,s,m)$ and $\rho(q,s,m)$ are the equilibrium strategies of the game without communication) constitutes a PBE.

play a given strategy constitutes an interval. Finally, we show that the equilibrium strategies admit a cutoff representation.

Step 1: *The equations characterizing responsive undominated PBE profiles* $((\mu, \sigma), \rho)$.

The sender's voting strategy. Given s and m, sequential rationality requires that $\sigma(q, s, m)$ admits the following cutoff structure:

$$\sigma(q, s, m) = \begin{cases} 1 \text{ if } q > \Pr(\omega = 0|s, m, v_r = 0) \\ 0 \text{ if } q < \Pr(\omega = 0|s, m, v_r = 0) \end{cases} \tag{3}$$

Since we are restricting attention to equilibria in undominated strategies, $\Pr(s, m, v_r = 0) > 0$. Thus $\Pr(\omega = 0|s, m, v_r = 0)$ is well defined. We define $q_{sm} = \Pr(\omega = 0|s, m, v_r = 0)$, where

$$q_{sm} = \frac{\Pr(s|\omega = 0)\Pr(v_r = 0|\omega = 0, m)}{\Pr(s|\omega = 0)\Pr(v_r = 0|\omega = 0, m) + \Pr(s|\omega = 1)\Pr(v_r = 0|\omega = 1, m)} \tag{4}$$

The receiver's voting strategy. Given s and m, sequential rationality requires that $\rho(q, s, m) = 1$, if

$$\begin{aligned} q[\Pr(s|\omega &= 0)\Pr(v_s = 0, m|\omega = 0) + \Pr(s|\omega = 1)\Pr(v_s = 0, m|\omega = 1)] > \\ \Pr(s|\omega &= 0)\Pr(v_s = 0, m|\omega = 0) \end{aligned} \tag{5}$$

and that $\rho(q, s, m) = 0$, if the inequality is reversed.

Whenever $\Pr(s|\omega = 0)\Pr(v_s = 0, m|\omega = 0) + \Pr(s|\omega = 1)\Pr(v_s = 0, m|\omega = 1) > 0$, the strategy $\rho(q, s, m)$ admits the following cutoff structure:

$$\rho(q, s, m) = \begin{cases} 1 \text{ if } q > \Pr(\omega = 0|s, m, v_s = 0) \\ 0 \text{ if } q < \Pr(\omega = 0|s, m, v_s = 0) \end{cases} \tag{6}$$

We define $r_{sm} = \Pr(\omega = 0|s, m, v_s = 0)$.

Whenever $\Pr(v_s = 0, m|\omega = 0) = 0$ (and thus $\Pr(v_s = 0, m|\omega = 1) = 0$), it follows that $d = 1$ regardless of $\rho(q, s, m)$. Since the receiver is indifferent between voting $v = 0$ and $v = 1$, we can construct a cutoff strategy for the receiver's voting choice that leaves the sender's incentives unchanged. Formally, we derive $\Pr(\omega = 0|s, m, v_r = 0, \rho)$ from the strategy $\rho(q, s, m)$. By the intermediate value theorem, there exists a $r_{sm} \in (0, 1)$ such that the cutoff strategy $\rho'(q, s, m)$ defined by r_{sm} yields $\Pr(\omega = 0|s, m, v_r = 0, \rho) = \Pr(\omega = 0|s, m, v_r = 0, \rho')$. This guarantees that the sender's voting and message strategies are unchanged.

The sender's message strategy. Consider a sender of type q at the message stage after she observes signal s. Let $Eu(m|s, q)$ denote her expected utility when sending message m. In equilibrium, the sender chooses $m = 1$ ($m = 0$) when the following function $\varphi(s,q)$ is positive (negative):

$$\begin{aligned}
&\varphi(s,q) := Eu(1|s,q) - Eu(0|s,q) \quad (7)\\
=\ & -q \Pr(d=0,\omega=1|s,q,m=1) - (1-q)\Pr(d=1,\omega=0|s,q,m=1)\\
& +q \Pr(d=0,\omega=1|s,q,m=0) + (1-q)\Pr(d=1,\omega=0|s,q,m=0)\\
=\ & -q \Pr(\omega=1|s)\left[\Pr(d=0|\omega=1,s,q,m=1) - \Pr(d=0|\omega=1,s,q,m=0)\right]\\
& -(1-q)\Pr(\omega=0|s)\left[\Pr(d=1|\omega=0,s,q,m=1) - \Pr(d=1|\omega=0,s,q,m=0)\right]\\
=\ & -q \Pr(\omega=1|s)\left[\chi(q<q_{s1})\Pr(v_r=0|\omega=1,m=1)\right.\\
& \left.-\chi(q<q_{s0})\Pr(v_r=0|\omega=1,m=0)\right]\\
& -(1-q)\Pr(\omega=0|s)\left[\chi(q<q_{s0})\Pr(v_r=0|\omega=0,m=0)\right.\\
& \left.-\chi(q<q_{s1})\Pr(v_r=0|\omega=0,m=1)\right]
\end{aligned}$$

where $\chi(.)$ denotes the indicator function.

Step 2: *Without loss of generality, we can restrict attention to equilibria in which, for a given signal s, the set of types who use the same strategy is an interval.*

For notational ease, we denote the strategies (μ,σ) by ξ and drop s. Consider a PBE (ξ,ρ), where there is a triple $q' < q < q''$, such that $\xi(q') = \xi(q'')$, but $\xi(q') \neq \xi(q)$. Perfection and equation (3) imply that it is impossible that $\sigma(q) \neq \sigma(q')$ Hence, it must be that $\mu(q) \neq \mu(q')$. Without loss of generality, we may assume that $\mu(q)$ and $\mu(q')$ are two different pure strategies. (If $\mu(q)$ or $\mu(q')$ is mixed, then there exists a PBE (ξ',ρ) in which the types q' and q use different pure message strategies, types q' and q'' use the same strategy, and the other types of sender maintain ξ.)

Since ξ is an equilibrium strategy, it follows that type q' prefers to play $\xi(q')$ rather than $\xi(q)$. This implies that:

$$\begin{aligned}
-q'\Pr(d= &\ 0,\omega=1|\xi(q')) - (1-q')\Pr(d=1,\omega=0|\xi(q')) \geq\\
-q'\Pr(d= &\ 0,\omega=1|\xi(q)) + (1-q')\Pr(d=1,\omega=0|\xi(q))
\end{aligned}$$

and

$$\begin{aligned}
-q''\Pr(d= &\ 0,\omega=1|\xi(q')) - (1-q'')\Pr(d=1,\omega=0|\xi(q')) \geq\\
-q''\Pr(d= &\ 0,\omega=1|\xi(q)) + (1-q'')\Pr(d=1,\omega=0|\xi(q)).
\end{aligned}$$

By linearity of the above expression, the strategy $\xi(q)$ is optimal for type q if and only if

$$\begin{aligned}
\Pr(d=0,\omega=1|\xi(q')) &= \Pr(d=0,\omega=1|\xi(q))\\
\Pr(d=1,\omega=0|\xi(q')) &= \Pr(d=1,\omega=0|\xi(q)) \quad (8)
\end{aligned}$$

which implies that the senders q, q' and q'' must be indifferent between $\xi(q)$ and $\xi(q')$. Since by equation (3) $\sigma(q) \in \{0,1\}$ a.e, it suffices to consider four different cases.

Case 1: Suppose that $\mu(q') \neq \mu(q)$ and that on-path $\sigma(q) = 0$ and $\sigma(q') = 1$. Then $\Pr(d = 0, \omega = 1|\xi(q')) = 0$ but $\Pr(d = 0, \omega = 1|\xi(q)) > 0$ since after any message the receiver may vote $v = 0$ with positive probability. This is a contradiction.

Case 2: Suppose that $\mu(q') \neq \mu(q)$ and that on-path $\sigma(q) = 1$ and $\sigma(q') = 0$. Then $\Pr(d = 0, \omega = 1|\xi(q')) > 0$ but $\Pr(d = 0, \omega = 1|\xi(q)) = 0$, which is again a contradiction.

Case 3: Suppose now that on-path $\sigma(q) = 0$ and $\sigma(q') = 0$. Using equation (8), we have

$$\begin{aligned} & \Pr\left(d = 1, \omega = 0|\xi\left(q'\right)\right) = \Pr\left(\omega = 0|s\right)\Pr\left(v_r = 1|\omega = 0, \mu\left(q'\right)\right) \qquad (9) \\ = \; & \Pr\left(\omega = 0|s\right)\left[p_r \Pr\left(v_r = 1|\mu\left(q'\right), s_r = 0\right) + (1 - p_r)\Pr\left(v_r = 1|\mu\left(q'\right), s_r = 1\right)\right] \\ = \; & \Pr\left(\omega = 0|s\right)\left[p_r \Pr\left(v_r = 1|\mu\left(q\right), s_r = 0\right) + (1 - p_r)\Pr\left(v_r = 1|\mu\left(q\right), s_r = 1\right)\right] \\ = \; & \Pr\left(d = 1, \omega = 0|\xi\left(q\right)\right) \end{aligned}$$

Note that if the set of sender's types who send message $\mu(q')$ and vote $v = 0$ on path has measure zero, then we can find a PBE (ξ', ρ) in which types $\hat{q} < q$ who were using strategy $\xi(q')$ switch to strategy $\xi'(\hat{q}) = \xi(q)$ and all other types of sender maintain strategy ξ. Similarly, if the set of sender's types who send message $\mu(q)$ and vote $v = 0$ on path has measure zero, then there exists a PBE in which all types $\hat{q} \in (q', q'')$ who were using strategy $\xi(q)$ adopt strategy $\xi(q')$. Finally, consider the case in which both sets defined above have positive measure. Then equation (5) implies that $r_{0\mu(q)} > r_{0\mu(q')}$ if and only if $r_{1\mu(q)} > r_{1\mu(q')}$. It follows that equation (9) can hold only if the equilibrium is not responsive.

Case 4: Suppose that on-path $\sigma(q) = 1$ and $\sigma(q') = 1$. Then there exists a PBE in which types $\hat{q} \in (q', q'')$ who were using strategy $\xi(q)$ adopt strategy $\xi(q')$, similarly to case 3.

This concludes the proof of the second step. We have shown that for any equilibrium, we can find an outcome-equivalent equilibrium where the set of types who take the same strategy is an interval. Moreover, the argument may be extended to show that if such an interval is of positive measure, the associated message strategy must be a pure strategy. If not, for any pair of type (q, q') in the interval, both q and q' must be indifferent between sending message $m = 0$ or $m = 1$. Also, given the same message, type q votes as type q'. Together with condition (7), this implies that the equilibrium must be non-responsive.

Step 3: *For any PBE in undominated strategies, there is an outcome-equivalent equilibrium in which the sender's message strategy can be characterized by a cutoff.*

Let Q_{mv} denote the interval of sender's types who send message m and vote v (on-path). Suppose there exists a PBE with $Q_{00} = (a, b)$ and $Q_{01} = (c, d)$, where $0 \leqslant a \leqslant b < c \leqslant d \leqslant 1$. Depending on the relationship of Q_{10} and Q_{11}, we consider three different cases.

Case 1: $Q_{10} = (b, e)$ and $Q_{11} = (e, c)$ for some $e \in [b, c)$.

Types $q \in Q_{01}$ are indifferent between sending message $m = 0$ and $m = 1$ because on-path they will vote $v = 1$ and achieve $d = 1$ in both cases. Therefore, there is an outcome-equivalent equilibrium where all types in Q_{01} adopt the same strategy as the types in Q_{11} and the receiver's strategy is unchanged.

Case 2: $Q_{10} = (b, c)$ and $Q_{11} = (d, 1)$.

There is an outcome-equivalent equilibrium where all types in Q_{11} adopt the same strategy as the types in Q_{01}, so this case is outcome-equivalent to $Q_{10} = (b, c)$ and $Q_{01} = (c, 1)$. It remains to show that there is an outcome-equivalent equilibrium where all the types in Q_{01} play $m = 1$ and $v = 1$. That is guaranteed when $q_{s1} \leq c$. Proceeding by contradiction, suppose $c < q_{s1}$. Note that $\varphi(s, q)$ defined in equation (7) must be equal to zero at $q = c$. Solving $\varphi(s, c) = 0$ yields an expression for c that coincides with the RHS of equation (4) with $m = 1$. This implies that $c = q_{s1}$, and a contradiction is obtained.

Case 3: $Q_{10} = (0, a)$ and $Q_{11} = (b, c)$.

There is an outcome-equivalent equilibrium where all types in Q_{01} adopt the same strategy as the types in Q_{11}, so this case is outcome-equivalent to $Q_{10} = (0, a)$ and $Q_{11} = (b, 1)$. To complete the argument, note that the reduced configuration derived in Case 3 is the meaning reversion of the reduced configuration derived in Case 2. ■

A.2 Robustness

Here we formalize the robustness requirement mentioned in the main body. For infinite games, Trembling Hand Perfection has been studied by Simon and Stinchcombe (1995), who distinguish between a strong and a weak extension of the standard concept introduced by Selten (1975) for finite games. Our requirement has the flavor of such extensions (applied to the agent normal form of our game) and imposes a further restriction. We require a robust cutoff equilibrium to be the limit of a sequence of cutoff strategy profiles, where all cutoffs are uniquely identified by sequential rationality, along some vanishing sequence of perturbations. The motivation for our refinement is that by inspecting the equations that characterize the equilibrium, we note that there are some cutoffs that may be undetermined. We then perturb the equilibrium strategies slightly and look for a perturbation where all the cutoffs are uniquely determined.

We characterize an agent in terms of her signal and, if applicable, also in terms of the message sent or received. Formally, the sender has six agents $z \in Z_s = \{0, 1\} \cup \{0, 1\}^2$ and the receiver has four agents $z \in Z_r = \{0, 1\}^2$. For every agent, we introduce a distributional strategy that assigns a random choice to every type q. For any $z_i \in Z_i$, $i \in \{s, r\}$, let the measurable function $\psi_{z_i} : (0, 1) \to \Delta(\{0, 1\})$ describe the distributional equilibrium strategy of agent z_i. Let $BR_q^{z_i}(\hat{\psi}_{-z_i})$ be the set of best-replies of type q of agent z_i against opponents' distributional strategy profile $\hat{\psi}_{-z_i}$. Let $BR^{z_i}(\hat{\psi}_{-z_i})$ be the set of distributional strategies induced by $BR_q^{z_i}(\hat{\psi}_{-z_i})$ with $q \in (0, 1)$.

Definition 1 *An equilibrium ψ is robust if there exist a sequence of positive numbers $\eta^n \to 0$ and a sequence $\{(\varepsilon^n_{z_i})_{Z_s \cup Z_r}\}_{n \geq 0}$ of measurable functions in $\dot{\Delta}(\{0,1\})^{(0,1)}$ (where $\dot{\Delta}$ is the interior of Δ) such that*

1. *for all $z_i \in Z_s \cup Z_r$, there exists a sequence $\{(\psi^n_{z_i})_{Z_s \cup Z_r}\}_{n \geq 0}$ where each $\psi^n_{z_i}$ is a selection from the set $BR^{z_i}((1-\eta^n)\psi^n_{-z_i} + \eta^n \varepsilon^n_{-z_i})$, $\psi^n_{z_i}$ admits cutoff representation, and it converges weakly to ψ_{z_i};*

2. *for any n and $z_i \in Z_s \cup Z_r$, the set $BR^{z_i}((1-\eta^n)\psi^n_{-z_i} + \eta^n \varepsilon^n_{-z_i})$ contains a unique strategy (up to sets of measure zero).*

We now show that in any robust cutoff PBE, all cutoffs are related across signals by the functions k_{p_s} and k_{p_r} as explained in Section 2.1.

Lemma 2 *In any robust cutoff PBE, $q_1 = k_{p_s}(q_0)$ and for any $m \in \{0,1\}$, $q_{1m} = k_{p_s}(q_{0m})$ and $r_{1m} = k_{p_r}(r_{0m})$.*

Proof. Our robustness requirement implies that along the sequence $\left(\psi^n_{z_i}\right)_{Z_s \cup Z_r}$ all cutoffs are uniquely determined by the equilibrium conditions. For example, the sender's voting cutoff after observing signal s and sending message m can be represented as

$$q^n_{sm} = \frac{1}{1 + \frac{\Pr(\omega=1|s)}{\Pr(\omega=0|s)}\left(\frac{B^n_m}{A^n_m}\right)}$$

for some strictly positive A^n_m and B^n_m. Therefore, $q^n_{1m} = k_{p_s}(q^n_{0m})$ for any n. Since the function k_p is continuous, $\lim_{n\to\infty} q^n_{1m} = k_{p_s}(\lim_{n\to\infty} q^n_{0m})$. A similar argument can be used to establish the relationship between the remaining cutoffs. ■

The concepts and results of this section extend *mutatis mutandis* to the two-sender game.

Appendix B: Proofs

In the proofs we shall often refer to the odds ratio $(1-p_i)/p_i$, which we denote by R_i. We will make use of the family of functions

$$k_R(q) = \frac{R^2 q}{R^2 q + 1 - q}$$

defined on $q \in (0,1)$ and indexed with $R \in (0,1)$. Note that for every q and R, $k_R(q) < q$ and k_R is strictly increasing in q. k_R is straightforwardly derived from k_p since $R = (1-p)/p$.

Proof of Proposition 1. We start by deriving the equations that characterize all robust responsive cutoff equilibria. We then check which order of the cutoffs is consistent with equilibrium. Specifically, we proceed in two steps. The first one rules out all configurations that are inconsistent with equilibrium. The second step shows that a robust cutoff PBE indeed exists for the configuration presented in Proposition 1.

Sender's voting strategy. After restricting attention to robust cutoff equilibria, equations (4), which characterize the sender's equilibrium voting strategy, simplify to:

$$\begin{cases} q_{00} = \frac{1}{1+R_s \frac{[R_r F(r_{00})+F(r_{10})]}{[F(r_{00})+R_r F(r_{10})]}} \\ q_{10} = \frac{1}{1+\frac{1}{R_s} \frac{[R_r F(r_{00})+F(r_{10})]}{[F(r_{00})+R_r F(r_{10})]}} \\ q_{01} = \frac{1}{1+R_s \frac{[R_r F(r_{01})+F(r_{11})]}{[F(r_{01})+R_r F(r_{11})]}} \\ q_{11} = \frac{1}{1+\frac{1}{R_s} \frac{[R_r F(r_{01})+F(r_{11})]}{[F(r_{01})+R_r F(r_{11})]}} \end{cases} \tag{10}$$

Receiver's voting strategy. Given m, r_{sm} is uniquely defined by equation (6) whenever $\Pr(v_s = 0, m) > 0$. In this case we obtain after simplification:

$$\begin{cases} r_{00} = \frac{1}{1+R_r \frac{[R_s \min\{F(q_0),F(q_{00})\}+\min\{F(q_1),F(q_{10})\}]}{[\min\{F(q_0),F(q_{00})\}+R_s \min\{F(q_1),F(q_{10})\}]}} \\ r_{10} = \frac{1}{1+\frac{1}{R_r} \frac{[R_s \min\{F(q_0),F(q_{00})\}+\min\{F(q_1),F(q_{10})\}]}{[\min\{F(q_0),F(q_{00})\}+R_s \min\{F(q_1),F(q_{10})\}]}} \end{cases} \tag{11}$$

$$\begin{cases} r_{01} = \frac{1}{1+R_r \frac{[R_s (F(q_{01})-F(q_0))+(F(q_{11})-F(q_1))]}{[(F(q_{01})-F(q_0))+R_s (F(q_{11})-F(q_1))]}} \\ r_{11} = \frac{1}{1+\frac{1}{R_r} \frac{[R_s (F(q_{01})-F(q_0))+(F(q_{11})-F(q_1))]}{[(F(q_{01})-F(q_0))+R_s (F(q_{11})-F(q_1))]}} \end{cases} \tag{12}$$

In a robust cutoff PBE, $\Pr(v_s = 0, m = 0) > 0$. Hence, r_{s0} is uniquely defined. If $\Pr(v_s = 0, m = 1) = 0$, then the characterizing equations for r_{s1} are undetermined. However, Lemma 2 in Appendix A implies that $r_{01} = k_{R_r}(r_{11})$ in a robust cutoff PBE.

Step 1: *The only configuration consistent with equilibrium is $q_s < q_{s0} < q_{s1}$ for $s \in \{0, 1\}$.*

Since cutoffs are related across signals by the strictly increasing functions k_{R_s} and k_{R_r}, it suffices to consider the ordering of the cutoffs for a given signal.

In any responsive cutoff equilibrium, we have $r_{s0} > r_{s1}$. (If $r_{s0} = r_{s1}$, the equilibrium is nonresponsive. Moreover, in any responsive equilibrium, we have $q_s > 0$. But if $r_{s0} < r_{s1}$, then one can show by inspecting equation (7) that there are senders of type q, with q sufficiently close to 0, who have an incentive to deviate from the equilibrium strategy and send message $m = 1$.) By equation (10) and our technical assumption, $r_{s0} > r_{s1}$ if and only if $q_{s0} < q_{s1}$. It remains to check configurations that involve $q_{s0} < q_{s1}$.

Case 1: Suppose that $q_{s0} < q_{s1} \leq q_s$. Then $\varphi(s, q) \leq 0$ for $q \in (q_{s0}, q_{s1})$ since these types prefer to send message $m = 0$. However, φ is strictly decreasing in this interval and has a zero at q_{s1} (see equation (10)). This implies a contradiction.

Case 2: Suppose we have an equilibrium where $q_{s0} < q_s < q_{s1}$. Under this configuration, the function $\varphi(s, q)$ in equation (7) is strictly decreasing for $q \in (q_{s0}, q_{s1})$ and $\varphi(s, q_s) = 0$. However, inspecting equation (10), we have $\varphi(s, q_{s1}) = 0$ which implies a contradiction.

Case 3: Suppose that $q_s = q_{s0} < q_{s1}$. We want to show that there are senders of type $q \in (0, q_{s0})$ who have an incentive to deviate from their equilibrium strategy and send $m = 1$. Consider the function φ from equation (7). This function is piecewise linear. We extend the segment defined on $(0, q_{s0})$ to the unit interval to obtain the following function:

$$\begin{aligned} \hat{\varphi}(s,q) &= -q \Pr(\omega = 1|s) \left[\Pr(v_r = 0|\omega = 1, m = 1) - \Pr(v_r = 0|\omega = 1, m = 0)\right] \\ &\quad - (1-q) \Pr(\omega = 0|s) \left[\Pr(v_r = 0|\omega = 0, m = 0) - \Pr(v_r = 0|\omega = 0, m = 1)\right] \end{aligned}$$

Note that $\hat{\varphi}(s,q)$ is strictly increasing with $\hat{\varphi}(s,0) < 0$ and $\hat{\varphi}(s,1) > 0$. Hence, it attains a unique zero in $(0,1)$. It thus suffices to show that $\hat{\varphi}(s,q)$ has its zero in $(0, q_{s0})$.

Let $\hat{q}_s$ be the zero of $\hat{\varphi}(s,q)$, where

$$\hat{q}_s = \frac{1}{1 + \frac{\Pr(\omega=1|s)[\Pr(v_r=0|\omega=1,m=0) - \Pr(v_r=0|\omega=1,m=1)]}{\Pr(\omega=0|s)[\Pr(v_r=0|\omega=0,m=0) - \Pr(v_r=0|\omega=0,m=1)]}}$$

From equation (4) we know that

$$q_{s0} = \frac{1}{1 + \frac{\Pr(\omega=1|s)\Pr(v_r=0|\omega=1,m=0)}{\Pr(\omega=0|s)\Pr(v_r=0|\omega=0,m=0)}}$$

A simple calculation shows that $\hat{q}_s \geq q_{s0}$ implies

$$\frac{\Pr(v_r = 0|\omega = 1, m = 0)}{\Pr(v_r = 0|\omega = 0, m = 0)} \leq \frac{\Pr(v_r = 0|\omega = 1, m = 1)}{\Pr(v_r = 0|\omega = 0, m = 1)}$$

But $q_{s1} > q_{s0}$ rules out the above inequality.

We conclude that a robust responsive cutoff PBE can exist only if $q_s < q_{s0} < q_{s1}$. The next step is to show that there indeed exists an equilibrium with this configuration.

Step 2: *There exists a robust responsive cutoff PBE.*

The following system characterizes the equilibrium with $q_s < q_{s0} < q_{s1}$:

$$\begin{cases} q_0 = \frac{1}{1+R_s \frac{[R_r(F(r_{00})-F(r_{01}))+(F(k_{R_r}(r_{00}))-F(k_{R_r}(r_{01})))]}{[(F(r_{00})-F(r_{01}))+R_r(F(k_{R_r}(r_{00}))-F(k_{R_r}(r_{01})))]}} \\ q_{00} = \frac{1}{1+R_s \frac{[R_r F(r_{00})+F(k_{R_r}(r_{00}))]}{[F(r_{00})+R_r F(k_{R_r}(r_{00}))]}} \\ q_{01} = \frac{1}{1+R_s \frac{[R_r F(r_{01})+F(k_{R_r}(r_{01}))]}{[F(r_{01})+R_r F(k_{R_r}(r_{01}))]}} \\ r_{00} = \frac{1}{1+R_r \frac{[R_s F(q_0)+F(k_{R_s}(q_0))]}{[F(q_0)+R_s F(k_{R_s}(q_0))]}} \\ r_{01} = \frac{1}{1+R_r \frac{[R_s(F(q_{01})-F(q_0))+(F(k_{R_s}(q_{01}))-F(k_{R_s}(q_0)))]}{[(F(q_{01})-F(q_0))+R_s(F(k_{R_s}(q_{01}))-F(k_{R_s}(q_0)))]}} \end{cases} \tag{13}$$

It is enough to spell out the equations for $s = 0$ because the cutoffs for $s = 1$ can be recovered from the cutoffs for $s = 0$ using the functions k_{R_r} and k_{R_s}.

Since q_{00} does not appear in the equations for the other cutoffs, it suffices to show that the system above admits a solution with $r_{00} > r_{01}$ and $q_{01} > q_0$. To see this, note that if $r_{00} > r_{01}$, then $\varphi(s, q)$ is strictly increasing for $q \in (0, q_{s0})$ and it has a zero at q_s. Moreover, $\varphi(s, q)$ is strictly decreasing for $q \in (q_{s0}, q_{s1})$ and has a zero at q_{s1}. For $q > q_{s1}$, $\varphi(s, q) = 0$. This implies that the cutoff message strategy described by q_s is a best reply. Finally, applying our technical assumption to the above system implies that if $q_{s1} > q_s$, then $q_{s0} \in (q_s, q_{s1})$.

For the moment consider the case in which $R_s = R_r = R$. We express the working hypothesis that there is a solution such that $r_{s0} = q_{s1}$ and $r_{s1} = q_s$. Therefore, we just need to find a solution that satisfies $q_{01} > q_0$ for the system:

$$\begin{cases} q_{01} = h_1(q_{01}, q_0) := \frac{1}{1+R\frac{[RF(q_0)+F(k_R(q_0))]}{[F(q_0)+RF(k_R(q_0))]}} \\ q_0 = h_2(q_{01}, q_0) := \frac{1}{1+R\frac{[R(F(q_{01})-F(q_0))+(F(k_R(q_{01}))-F(k_R(q_0)))]}{[(F(q_{01})-F(q_0))+R(F(k_R(q_{01}))-F(k_R(q_0)))]}} \end{cases}$$

Consider the function $h = (h_1, h_2)$ defined on $X = \{(x, y) \in (0, 1)^2 : x > y\}$. For any $(x, y) \in X$, our technical assumption implies that $h_1(x, y) > h_2(x, y)$. Moreover, $h_1(x, y) \in (0, 1)$ and $h_2(x, y) \in (0, 1)$, so the function h maps X into X.

Denote by $\bar{X}$ the closure of X. We now construct a continuous extension $\tilde{h} : \bar{X} \to \bar{X}$ of h. First note that $\tilde{h}_1(x, y) = h_1(x, y)$ is a continuous function on $\bar{X} \backslash \{(x, 0) : x \in [0, 1]\}$. Using De L'Hopital rule, we define $\tilde{h}_1(x, 0) = \lim_{y \to 0} h_1(x, y)$ for any x. (Recall that $h_1(x, y)$ is independent of x. Hence, $\lim_{y \to 0} h_1(x, y)$ is independent of x.) Secondly, $\tilde{h}_2(x, y) = h_2(x, y)$ is a continuous function on $\bar{X} \backslash \{(y, y) : y \in [0, 1]\}$. We again use De L'Hopital rule to define

$$\tilde{h}_2(y, y) = \lim_{x \to y} h_2(x, y) = \frac{1}{1 + R\frac{[Rf(y)+f(k(y))k'(y)]}{[f(y)+Rf(k(y))k'(y)]}}$$

for any y.

$\tilde{h}$ is continuous. Our technical assumption implies that $\tilde{h}_2(y, y) < \tilde{h}_1(y, y)$ for any y. Moreover, $\tilde{h}_1(x, 0) \in (0, 1)$, $\tilde{h}_2(x, 0) \in (0, 1)$, and $\tilde{h}_2(1, y) \in (0, 1)$. This implies that $\tilde{h} : \bar{X} \to X \subset \bar{X}$. Hence, by Brouwer's Fixed-Point Theorem, there must exist a pair $(x, y) \in X$ such that $h(x, y) = \tilde{h}(x, y) = (x, y)$.

When $R_s \neq R_r$, it is no longer the case that $r_{s0} = q_{s1}$ and $r_{s1} = q_s$. Nevertheless, the above argument can be generalized by considering a function $h = (h_1, h_2, h_3, h_4)$ on $X = \{(x, y, w, v) \in (0, 1)^4 : x > y, w > v\}$, where h_j denotes the RHS of the jth equation in system (13).

Since all the cutoffs are uniquely determined in equilibrium and all the characterizing equations are continuous, the equilibrium must be robust. ■

Proof of Corollary 1. We show that $R_s = R_r = R$ implies $q_{11} < 1/2 < q_0$ and

$r_{10} < 1/2 < r_{01}$. To show this it suffices to demonstrate that the smallest cutoff for $s = 0$ is larger than the largest cutoff for $s = 1$.

Each cutoff associated with signal $s = 0$ has a representation

$$\frac{1}{1 + R\frac{\Pr(\omega=1)[RA+B]}{\Pr(\omega=0)[A+RB]}}$$

where $A, B \in (0,1)$. Such an expression is strictly larger than $\frac{1}{1+\frac{\Pr(\omega=1)}{\Pr(\omega=0)}}$, the lower bound obtained by setting $A = 0$ and $B = 1$. Each cutoff associated with signal $s = 1$ has a representation

$$\frac{1}{1 + \frac{1}{R}\frac{\Pr(\omega=1)[RA+B]}{\Pr(\omega=0)[A+RB]}}$$

Such an expression is strictly smaller than $\frac{1}{1+\frac{\Pr(\omega=1)}{\Pr(\omega=0)}}$, the upper bound obtained by setting $A = 1$ and $B = 0$. ■

Proof of Proposition 2. Let q_s, q_{sm}, and r_{sm} be an arbitrary profile of equilibrium cutoffs of the game $\Gamma(p, p')$. By inspecting the equations in system (13) it is easy to check that $\Gamma(p', p)$ admits a responsive equilibrium with cutoffs q'_s, q'_{sm}, and r'_{sm} satisfying:

$$q'_s = r_{s1}, \quad q'_{s1} = r_{s0}, \quad r'_{s1} = q_s, \quad r'_{s0} = q_{s1} \quad \text{for } s = 0, 1. \tag{14}$$

Denote by $u_s(p, p')$ the sender's ex-ante utility associated with the profile (q_s, q_{sm}, r_{sm}) and by $u_r(p, p')$ the receiver's ex-ante utility. Similarly, denote by $u_s(p', p)$ and $u_r(p', p)$ the sender's and receiver's ex-ante utility associated with the profile (q'_s, q'_{sm}, r'_{sm}). Straightforward calculations yield:

$$\begin{aligned}
u_s(p, p') = \; & \frac{1}{2}\{\int_0^{q_0} [p((1-p')(1-F(r_{10})) + p'(1-F(r_{00})))(q-1) \\
& +(1-p)(p'F(r_{10}) + (1-p')F(r_{00}))(-q)]f(q)\,dq \\
& +\int_{q_0}^{q_{01}} [p((1-p')(1-F(r_{11})) + p'(1-F(r_{01})))(q-1) \\
& +(1-p)(p'F(r_{11}) + (1-p')F(r_{01}))(-q)]f(q)\,dq + \int_{q_{01}}^{1} p(q-1)f(q)\,dq \\
& +\int_0^{q_1} [(1-p)((1-p')(1-F(r_{10})) + p'(1-F(r_{00})))(q-1) \\
& +p(p'F(r_{10}) + (1-p')F(r_{00}))(-q)]f(q)\,dq \\
& +\int_{q_1}^{q_{11}} [(1-p)((1-p')(1-F(r_{11})) + p'(1-F(r_{01})))(q-1)
\end{aligned}$$

$$+p(p'F(r_{11})+(1-p')F(r_{01}))(-q)]f(q)\,dq+\int_{q_{11}}^{1}(1-p)(q-1)f(q)\,dq\}$$

$$\begin{aligned} u_r(p',p) &= \frac{1}{2}\{\int_0^{r'_{01}}[p((1-p')(1-F(q'_{11}))+p'(1-F(q'_{01})))(q-1) \\ &+(1-p)(p'F(q'_{11})+(1-p')F(q'_{01}))(-q)]f(q)\,dq \\ &+\int_{r'_{01}}^{r'_{00}}[p((1-p')(1-F(q'_{1}))+p'(1-F(q'_{0})))(q-1) \\ &+(1-p)(p'F(q'_{1})+(1-p')F(q'_{0}))(-q)]f(q)\,dq+\int_{r'_{00}}^{1}p(q-1)f(q)\,dq \\ &+\int_0^{r'_{11}}[(1-p)((1-p')(1-F(q'_{11}))+p'(1-F(q'_{01})))(q-1)+p(p'F(q'_{11}) \\ &+(1-p')F(q'_{01}))(-q)]f(q)\,dq \\ &+\int_{r'_{11}}^{r'_{10}}[(1-p)((1-p')(1-F(q'_{1}))+p'(1-F(q'_{0})))(q-1) \\ &+p(p'F(q'_{1})+(1-p')F(q'_{0}))(-q)]f(q)\,dq+\int_{r'_{10}}^{1}(1-p)(q-1)f(q)\,dq\} \end{aligned}$$

The equality $u_s(p,p')=u_r(p',p)$ follows by substituting equations (14) in the above expressions. By the same token, the equality $u_r(p,p')=u_s(p',p)$ is established. ■

Proof of Proposition 3. As indicated in Appendix A, in any robust PBE the order of the cutoffs is the same for $s=0$ and $s=1$ since cutoffs are related across signals by the strictly increasing function k_R. This allows us to focus on the cutoffs for one of the two signals. The proof proceeds in a number of steps. We first rule out all configurations which are inconsistent with an equilibrium and then show that the remaining three configurations admit an equilibrium.

In the remainder of the proof we shall make use of the following equations. Let V denote the opponent's vote. Consider the message cutoff first. It is optimal for a player of type q to send $m=1$ if $\phi(s,q)=Eu(1|s,q)-Eu(0|s,q)\geq 0$, where

$$\begin{aligned} &\phi(s,q)=-q\Pr(\omega=1|s)\left[\chi(q<q_{s10})\Pr(V=0,M=0|\omega=1,m=1)\right. \\ &-\chi(q<q_{s00})\Pr(V=0,M=0|\omega=1,m=0) \\ &+\chi(q<q_{s11})\Pr(V=0,M=1|\omega=1,m=1) \\ &-\chi(q<q_{s01})\Pr(V=0,M=1|\omega=1,m=0)] \\ &-(1-q)\Pr(\omega=0|s)\left[\chi(q<q_{s00})\Pr(V=0,M=0|\omega=0,m=0)\right. \\ &-\chi(q<q_{s10})\Pr(V=0,M=0|\omega=0,m=1) \\ &+\chi(q<q_{s01})\Pr(V=0,M=1|\omega=0,m=0) \end{aligned} \tag{15}$$

$$-\chi\left(q<q_{s11}\right)\Pr\left(V=0,M=1|\omega=0,m=1\right)]$$

Consider the voting cutoffs next. In a cutoff PBE, there is a strictly positive probability that a player is pivotal after receiving message $M=0$. Therefore, the voting cutoffs for $s\in\{0,1\}$ and $m\in\{0,1\}$ are uniquely identified and equal to

$$q_{sm0}=\frac{1}{1+\frac{\Pr(\omega=1|s)}{\Pr(\omega=0|s)}\frac{[R\min\{F(q_0),F(q_{00m})\}+\min\{F(k_R(q_0)),F(k_R(q_{00m}))\}]}{[\min\{F(q_0),F(q_{00m})\}+R\min\{F(k_R(q_0)),F(k_R(q_{00m}))\}]}}:=g_1(\min\{q_s,q_{s0m}\}) \tag{16}$$

On the other hand, a player may not be pivotal after observing message $M=1$. However, if there is a positive probability that the player is pivotal, then the voting cutoffs for $s\in\{0,1\}$ and $m\in\{0,1\}$ are uniquely identified and equal to

$$q_{sm1}=\frac{1}{1+\frac{\Pr(\omega=1|s)}{\Pr(\omega=0|s)}\frac{[R(F(q_{01m})-F(q_0))+(F(k_R(q_{01m}))-F(k_R(q_0)))]}{[(F(q_{01m})-F(q_0))+R(F(k_R(q_{01m}))-F(k_R(q_0)))]}}:=g_2(q_{s1m},q_s) \tag{17}$$

Our technical assumption guarantees that g_1 is strictly decreasing in its argument and that g_2 is strictly decreasing in both its arguments. (Note that g_2 is defined only for $q_{s1m}\neq q_s$.)

Step 1: *In any responsive robust equilibrium,* $q_s<q_{s10}$.

Inspecting equation (15), we note that a necessary condition for equilibrium is that for at least one signal $s\in\{0,1\}$ we have

$$\begin{aligned}&\min\{F(q_s),F(q_{s00})\}+\max\{F(q_{s10})-F(q_s),0\}\\ \geq\ &\min\{F(q_s),F(q_{s01})\}+\max\{F(q_{s11})-F(q_s),0\}\end{aligned} \tag{18}$$

Otherwise a player with type q sufficiently close to 0 would strictly prefer to send the message $m=1$.

Suppose by contradiction that $q_{s10}\leq q_s$. Equation (18) becomes

$$\min\{F(q_s),F(q_{s00})\}\geq\min\{F(q_s),F(q_{s01})\}+\max\{F(q_{s11})-F(q_s),0\} \tag{19}$$

We distinguish two cases.

Case 1: $q_{s01}\geq q_s$. Equation (19) implies that $q_{s00}\geq q_s$ and $q_{s11}\leq q_s$. Equation (16) then yields $q_{s10}=q_s=q_{s00}$. Wrapping up we obtain $q_{s11}\leq q_{s10}=q_s=q_{s00}\leq q_{s01}$. While this configuration allows for equilibria, these equilibria are outcome-equivalent to equilibria of the game without communication. Specifically, whenever $q>q_s$, the player will choose $m=1$ and $v=1$ irrespective of her opponent's message and whenever $q<q_s$, the player will always choose $m=0$ and vote $v=0$.

Case 2: $q_{s01}<q_s$. Equation (16) implies that $q_{s00}<q_s$. Otherwise we would have $q_{s00}=g_1(q_s)$ and $q_{s10}=g_1(q_{s01})$. But since g_1 is decreasing, $q_{s00}<q_{s10}\leq q_s$ and we would then have a contradiction.

Equation (18) becomes $F(q_{s00})\geq F(q_{s01})+\max\{F(q_{s11})-F(q_s),0\}$. Thus $q_{s00}\geq$

q_{s01}. Together with $q_s > q_{s00}$ this implies $q_{s10} = g_1(q_{s01})$ and $q_{s00} = g_1(q_{s00})$. Since g_1 is strictly decreasing, we obtain $q_{s10} \geq q_{s00}$. We need to distinguish two subcases.

Subcase 1: $q_{s00} = q_{s10}$. Thus $q_{s01} = q_{s10} = q_{s00} < q_s$. Since $q_{s01} = q_{s00}$, $\max\{F(q_{s11}) - F(q_s), 0\} = 0$, i.e., $q_{s11} \leq q_s$. While equilibria exist in this configuration, they are outcome-equivalent to the equilibria of the pure voting game. Whenever $q < q_{s00}$, the player will choose $m = 0$ and $v = 0$ independently of her opponent's message and whenever $q > q_{s00}$, the player will always vote $v = 1$.

Subcase 2: $q_{s00} < q_{s10}$. Since $q_{s10} = g_1(q_{s01})$ and $q_{s00} = g_1(q_{s00})$, this implies $q_{s01} < q_{s00}$. Suppose first that $q_{s11} > q_s$. For $q > q_{s10}$ the function $\phi(s, q)$ is strictly decreasing and has a zero at q_{s11}. Therefore the configuration $q_{s01} < q_{s00} < q_{s10} < q_s < q_{s11}$ does not constitute an equilibrium since types $q \in (q_{s10}, q_s)$ prefer to deviate from their equilibrium message strategy and send $m = 1$. Suppose next that $q_{s11} \leq q_s$. For $q \in (q_{s00}, q_{s10})$ the function $\phi(s, q)$ is strictly decreasing and has a zero at q_{s10}. Thus the configuration $q_{s01} < q_{s00} < q_{s10} < q_s$ and $q_{s11} \leq q_s$ is not an equilibrium since types $q \in (q_{s00}, q_{s10})$ have an incentive to deviate and send message $m = 1$.

Step 2: *In any responsive robust equilibrium,* $q_{s00} \leq q_{s10}$.

Suppose by contradiction that $q_{s00} > q_{s10}$. Then $q_{s00} > q_{s10} > q_s$ by Step 1. Hence, $q_{s00} = g_1(q_s)$ and $q_{s10} = g_1(\min\{q_s, q_{s01}\})$ which implies $q_{s00} \leq q_{s10}$ since g_1 is decreasing.

Step 3: *In any responsive robust equilibrium,* $q_s \leq q_{s00}$.

By contradiction, suppose that $q_{s00} < q_s$. Then $q_{s00} < q_s < q_{s10}$ by Step 1. Since $q_{s00} = g_1(q_{s00})$ and $q_{s10} = g_1(\min\{q_s, q_{s01}\})$, the previous inequality implies $q_{s01} < q_{s00}$. So we obtain that $q_{s01} < q_{s00} < q_s < q_{s10}$. This leaves us with q_{s11}. We now show that there does not exist a robust PBE irrespective of q_{s11}.

Suppose first that $q_{s11} < q_s$. In the interval $(\max\{q_{s00}, q_{s01}, q_{s11}\}, q_{s10})$ the function $\phi(s, q)$ is strictly decreasing and it has a zero at q_{s10}. Therefore types in the interval $(\max\{q_{s00}, q_{s01}, q_{s11}\}, q_s)$ have an incentive to deviate from the candidate equilibrium strategy and send message $m = 1$. Next assume that $q_s = q_{s11}$. In the interval (q_{s00}, q_{s10}) the function $\phi(s, q)$ is strictly decreasing and it has a zero at q_{s10}. Therefore types $q \in (q_{s00}, q_s)$ deviate by sending message $m = 1$. Finally suppose that $q_{s11} > q_s$. In the interval $(q_{s00}, \min\{q_{s11}, q_{s10}\})$ the function $\phi(s, q)$ is strictly decreasing. Hence, it cannot be negative on (q_{s00}, q_s) and positive on $(q_s, \min\{q_{s11}, q_{s10}\})$ and there cannot be an equilibrium.

To summarize, so far we have concluded that

$$q_s \leq q_{s00} \leq q_{s10} \quad \text{and} \quad q_s < q_{s10}.$$

This allows for three possibilities: q_{s01} may be strictly smaller, equal, or strictly larger than q_s. In the remainder of the proof we show that the only equilibrium configuration in which $q_{s01} > q_s$ is class 0, the only equilibrium configuration with $q_{s01} < q_s$ is class 1, and the only one with $q_{s01} = q_s$ is class 2.

Step 4: *The only robust responsive equilibrium configuration with $q_{s01} > q_s$ is class 0.*

Since $q_{s01} > q_s$, we have $q_{s00} = g_1(q_s) = q_{s10}$. The relationship between q_{s00} and q_{s01} determines three cases.

Case 1: $q_{s10} = q_{s00} < q_{s01}$. Equation (18) implies that $q_{s10} \geq q_{s11}$. Therefore, the function $\varphi(s, q)$ is strictly increasing in the interval (q_{s10}, q_{s01}) and it has a zero at q_{s01}. Consequently, types $q \in (q_{s00}, q_{s01})$ deviate from the candidate equilibrium strategy and send message $m = 0$.

Case 2: $q_{s10} = q_{s00} = q_{s01} > q_s$. This implies $q_{s00} = g_1(q_s) = g_2(0, q_s)$ and $q_{s01} = g_2(q_{s10}, q_s)$. Since g_2 is strictly decreasing in the first argument and $q_{s10} > q_s$, it follows that $q_{s00} > q_{s01}$, a contradiction.

Case 3: $q_{s10} = q_{s00} > q_{s01} > q_s$. There are four possibilities determined by the position of q_{s11}. Suppose first that $q_{s11} \leq q_s$. In the interval (q_s, q_{s01}) the function $\phi(s, q)$ is strictly increasing and it has a zero at q_{s01}. Types in this interval thus want to deviate from the candidate equilibrium strategy and send message $m = 0$. Assume next that $q_{s11} \in (q_s, q_{s01}]$. Since $q_{s01} = g_2(q_{s10}, q_s)$ and $q_{s11} = g_2(q_{s11}, q_s)$ and g_2 is strictly decreasing in its first argument, this would imply $q_{s01} < q_{s11}$, a contradiction. Finally, suppose that $q_{s11} \geq q_{s10}$. Since $q_{s11} = g_2(q_{s11}, q_s) \leq g_2(q_{s10}, q_s) = q_{s01} < q_{s10}$, we obtain a contradiction.

We are left with the configuration

$$q_s < q_{s01} < q_{s11} < q_{s00} = q_{s10}$$

All these cutoffs are uniquely identified by equations (15) and (16).[10] This candidate is in fact an equilibrium configuration and we denote it as class 0. By inspecting the $\phi(s, q)$ function, we observe that it is strictly increasing and it has a zero on $(0, q_{s01})$. This in turn identifies q_s. $\phi(s, q)$ is strictly decreasing on (q_{s01}, q_{s11}), it has a zero at q_{s11}, and it is constant and equal to zero for $q \geq q_{s11}$. In order to show the existence of an equilibrium of class 0, it suffices to show that the system of equations (15) and (16) admits a solution that satisfies the above configuration. The existence proof is a straightforward extension of the existence proof for one-sender game and is thus omitted.

To show that the equilibrium is robust, note that all voting cutoff are uniquely determined in equilibrium, but the function $\phi(s, q)$ is flat and equal to zero for $q \geq q_{s11}$. Assign the tremble such that with a small probability the players must vote $v = 0$ after sending message $m = 1$ irrespective of the received message. That amounts to mixing the original $\phi(s, q)$ with the extension to $\mathbb{R}$ of the segment of $\phi(s, q)$ that belongs on $(0, q_{s01})$. Since the latter is strictly increasing and larger than zero, the mix is larger than zero, the indeterminacy is solved to get $q_s = q_{s01}$. The mix does not modify the incentives on any other segment of the original $\phi(s, q)$. Symmetry allows us to conclude that the equilibrium is robust.

[10]This implies that all cutoffs associated with signal $s = 0$ ($s = 1$) are above (below) 1/2.

Step 5: *The only responsive robust equilibrium condition in which* $q_{s01} < q_s$ *is class* 1.

Note that $q_{s01} < q_s$ implies $q_{s00} < q_{s10}$. Moreover, we have $q_{s11} < q_{s10}$. Otherwise, since $q_{s11} = g_2(q_{s11}, q_s) \leq g_2(q_{s10}, q_s) = q_{s01} < q_{s10}$, we would obtain a contradiction. The relationship between q_s and q_{s00} yields two cases.

Case 1: $q_{s00} = q_s$. Depending on the position of q_{s11} there are two possibilities. First assume that $q_{s11} \leq q_s$. In the interval (q_{s01}, q_{s00}) the function $\phi(s, q)$ is strictly increasing and it has a zero at $\hat{q} = g_2(q_s, q_{s01}) < g_2(q_s, 0) = g_1(q_s) = q_{s00}$. Therefore, types in $(\max\{\hat{q}, q_{s01}\}, q_{s00})$ prefer to deviate at the message stage and send $m = 1$. Assume next that $q_{s11} \in (q_s, q_{s10})$. We have $q_{s11} = g_2(q_{s11}, q_s) < g_2(0, q_s) = g_1(q_s) = q_{s00} = q_s$, a contradiction.

Case 2: $q_{s00} > q_s$. There are three possibilities depending on the position of q_{s11}. First suppose that $q_{s11} \in [q_{s00}, q_{s10})$. We have $q_{s11} = g_2(q_{s11}, q_s) < g_2(0, q_s) = g_1(q_s) = q_{s00}$, a contradiction. Suppose next that $q_{s11} \in (q_s, q_{s00})$. For $q \in (q_{s11}, q_{s00})$ the function $\phi(s, q)$ is strictly increasing and it has a zero at $\hat{q} = g_2(q_s, q_{s01}) > g_2(q_s, q_{s11}) = q_{s11}$. Therefore, types in $(q_{s11}, \min\{\hat{q}, q_{s00}\})$ prefer to deviate at the message stage and send $m = 0$.

We are left with the configuration

$$q_{s01} < q_s < q_{s00} < q_{s10} \quad \text{and} \quad q_{s11} \leq q_s$$

This is the equilibrium configuration which we denote as class 1. The function $\phi(s, q)$ is strictly increasing and always negative on $(0, q_{s01})$ and strictly increasing on (q_{s01}, q_{s00}) with a zero in the interior of that interval. Moreover, $\phi(s, q)$ is strictly decreasing and always positive on (q_{s00}, q_{s10}) and it is zero for any $q \geq q_{s10}$. To show that the system of equations (15) and (16) has a solution that respects the above configuration is again straightforward extension of the proof for the one-sender game.

To show that the equilibrium is robust, notice that the voting cutoff q_{s11} is undetermined. However, weak dominance restricts the domain of definition to $q_{011} \in [1/2, q_0]$ and $q_{111} \in [0, q_1]$. A different perturbation should thus be derived for any couple (q_{011}, q_{111}). However, since q_{s11} is outcome-irrelevant, if we find one such couple, then we know that any other robust equilibrium is outcome-equivalent. Consider the perturbation in the voting stage where each player votes $v = 0$ with small probability after sending $m = 1$ and receiving $M = 1$. Along the perturbation, we have

$$q_{s11} = \frac{1}{1 + \frac{\Pr(s|\omega=1)}{\Pr(s|\omega=0)} \frac{\Pr(m=1|\omega=1)}{\Pr(m=1|\omega=0)}} = g_2(1, q_s),$$

and $q_s = g_2(q_s, q_{s01}) > g_2(1, q_s) = q_{s11}$ because $q_s < 1$ and $q_{s01} < q_s$. It remains to consider the message cutoffs. The function $\phi(s, q)$ is flat and equal to zero only when $q \geq q_{s10}$. We can take a perturbation that makes the player vote $v = 0$ with small probability if and only if she received the message $M = 0$ (regardless of the message sent). That amounts to mixing the original $\phi(s, q)$ with the extension to $\mathbb{R}$ of the segment of $\phi(s,q)$ that belongs on (q_{s01}, q_{s00}), which is strictly increasing and above zero on $(q_{s10}, 1)$, thus assuring the mixture to be always positive. The mix does not modify the incentives on any other segment of the original $\phi(s,q)$. The argument is completed by introducing a perturbation that consists of a mixture of the two different perturbations with weight on the first perturbation which is infinitesimal with respect to the weight placed on the second one.

Step 6: *The only responsive robust equilibrium condition in which $q_{s01} = q_s$ is class 2.*

The equality $q_{s01} = q_s$ immediately implies $q_{s00} = g_1(q_s) = g_1(q_{s01}) = q_{s10}$. Moreover, we have already shown that $q_{s10} > q_s$. Depending on the position of q_{s11} there are three cases to consider.

Case 1: $q_{s11} \geq q_{s10}$. Then $q_{s11} = g_2(q_{s11}, q_s) \leq g_2(q_{s10}, q_s) = q_{s01} < q_{s10}$, a contradiction.

Case 2: $q_{s11} \in (q_s, q_{s10})$. In the interval $(0, q_s)$ the function $\phi(s,q)$ is strictly increasing function and it has a zero at $\hat{q} = g_2(q_{s10}, q_{s11}) < g_2(q_s, q_{s11}) = q_s$. Hence, types in $(\hat{q}, q_s)$ deviate from the equilibrium by sending $m = 1$.

Case 3: We are left with the configuration

$$q_{s11} \leq q_s = q_{s01} < q_{s00} = q_{s10}$$

This is the equilibrium configuration we define as class 2. The function $\phi(s,q)$ is strictly increasing and always negative on $(0, q_{s01})$ and is identically equal to zero for $q \geq q_{s01}$. The existence proof is identical to the existence proof for the one-sender game after setting $R_s = R_r = R$, imposing $r^1_{s0} = q^1_{s1}$ and $r^1_{s1} = q^1_s$, and relabeling $q_{s01} = q^1_s$ and $q_{s10} = q^1_{s1}$, where the superscript 1 denotes the one-sender game cutoffs.

To show robustness, for the voting stage proceed as for the class 1 equilibrium and notice that $q_s = q_{s01} = g_2(q_{s10}, q_s) > g_2(1, q_s) = q_{s11}$ because $q_{s10} < 1$. With respect to the message cutoffs, the function $\phi(s,q)$ is flat and equal to zero only when $q \geq q_{s01}$. We can take a perturbation that makes the player always vote $v = 0$ with small probability. That amounts to mixing the original $\phi(s,q)$ with the extension to $\mathbb{R}$ of the segment of $\phi(s,q)$ that belongs on $(0, q_{s01})$, which is strictly increasing and above zero on $(q_{s01}, 1)$, thus assuring the mixture to be always positive. As for the class 1 equilibrium, the argument is then completed by taking a suitable mixture of perturbations.

Finally, we show that equilibria of class 0 are outcome-equivalent to equilibria of class 1. Given a class 0 equilibrium (q^0_s, q^0_{smM}), it is easy to show by inspecting the equations that characterize the equilibria that there exists an class 1 equilibrium (q^1_s, q^1_{smM}) such that

$$q^0_s = q^1_{s01}, \quad q^0_{s11} = q^1_s, \quad q^0_{s10} = q^1_{s10}$$

The above equalities and the fact the the cutoffs q^1_{s11}, q^1_{s00}, and q^0_{s01} are irrelevant for the outcome imply that the two equilibria are outcome-equivalent. In the same way, given a class 1 equilibrium, it is possible to construct an outcome-equivalent class 0 equilibrium. The details of the proof are available upon request. ■

Proof of Proposition 4. Available upon request. ■

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Proof of Lemma 1. In any babbling equilibrium both players use a cutoff voting strategy. The cutoffs of the sender (q_{sm}) and those of the receiver (r_{sm}) are given by:

$$\begin{cases} q_{sm} = \frac{1}{1+\frac{\Pr(s|\omega=1)\Pr(v_r=0|\omega=1)}{\Pr(s|\omega=0)\Pr(v_r=0|\omega=0)}} \\ r_{sm} = \frac{1}{1+\frac{\Pr(s|\omega=1)\Pr(v_s=0|\omega=1)}{\Pr(s|\omega=0)\Pr(v_s=0|\omega=0)}} \end{cases} \tag{20}$$

Consider now a non-responsive equilibrium in which the receiver is pivotal with positive probability after both messages. The equations characterizing the voting cutoffs are:

$$\begin{cases} q_{sm} = \frac{1}{1+\frac{\Pr(s|\omega=1)\Pr(v_r=0|m,\omega=1)}{\Pr(s|\omega=0)\Pr(v_r=0|m,\omega=0)}} \\ r_{sm} = \frac{1}{1+\frac{\Pr(s|\omega=1)\Pr(v_s=0,m|\omega=1)}{\Pr(s|\omega=0)\Pr(v_s=0,m|\omega=0)}} \end{cases} \tag{21}$$

In any non-responsive equilibrium $r_{s0} = r_{s1}$ and therefore:

$$\frac{\Pr(v_s = 0, m = 0|\omega = 1)}{\Pr(v_s = 0, m = 0|\omega = 0)} = \frac{\Pr(v_s = 0, m = 1|\omega = 1)}{\Pr(v_s = 0, m = 1|\omega = 0)} =: x$$

This implies that

$$\frac{\Pr(v_s = 0|\omega = 1)}{\Pr(v_s = 0|\omega = 0)} = \frac{\Pr(v_s = 0, m = 0|\omega = 1) + \Pr(v_s = 0, m = 1|\omega = 1)}{\Pr(v_s = 0, m = 0|\omega = 0) + \Pr(v_s = 0, m = 1|\omega = 0)} = x.$$

Also $r_{s0} = r_{s1}$ implies that $\Pr(v_r = 0|m = 0, \omega) = \Pr(v_r = 0|m = 1, \omega)$ and therefore

$$\frac{\Pr(v_r = 0|m = 0, \omega = 1)}{\Pr(v_r = 0|m = 0, \omega = 0)} = \frac{\Pr(v_r = 0|m = 1, \omega = 1)}{\Pr(v_r = 0|m = 1, \omega = 0)} = \frac{\Pr(v_r = 0|\omega = 1)}{\Pr(v_r = 0|\omega = 0)}.$$

This implies that any solution to system (21) also solves the system (20). Hence in both equilibria the sender and the receiver adopt the same voting strategy.

Finally, suppose that the receiver is not pivotal after some message m. In this case the cutoffs associated with the non-responsive equilibrium are found by solving system (21) without considering the equations for r_{sm}. In this case we have $\Pr(v_s = 0, m|\omega = 1) = \Pr(v_s = 0, m|\omega = 0) = 0$ which implies that for $m' \neq m$,

$$\frac{\Pr(v_s = 0, m'|\omega = 1)}{\Pr(v_s = 0, m'|\omega = 0)} = \frac{\Pr(v_s = 0|\omega = 1)}{\Pr(v_s = 0|\omega = 0)}$$

Again, any solution to system (21) is also a solution to system (20). ■

Lemma 3 *In the two-sender game, any symmetric non-responsive equilibrium is outcome-equivalent to a symmetric babbling equilibrium.*

Proof. Let V denote the opponent's vote. In any babbling equilibrium both players use a cutoff voting strategy. All cutoffs are given by the following expression:

$$q_{smM} = \frac{1}{1 + \frac{\Pr(s|\omega=1)\Pr(V=0|\omega=1)}{\Pr(s|\omega=0)\Pr(V=0|\omega=0)}} \tag{22}$$

Consider now a non-responsive equilibrium. Since a player must be pivotal after some message, assume without loss of generality that the pair (q_{s00}, q_{s10}) is uniquely identified by the equilibrium equations. Moreover, since in a non-responsive equilibrium, $q_{sm0} = q_{sm1}$ for both messages, all voting cutoffs are uniquely identified. The equation characterizing the voting cutoffs is:

$$q_{sm0} = \frac{1}{1 + \frac{\Pr(s|\omega=1)\Pr(V=0,M=0|m,\omega=1)}{\Pr(s|\omega=0)\Pr(V=0,M=0|m,\omega=0)}} \tag{23}$$

We distinguish two cases. If $\Pr(V = 0, M = 1|m) > 0$, then

$$q_{sm1} = \frac{1}{1 + \frac{\Pr(s|\omega=1)\Pr(V=0,M=1|m,\omega=1)}{\Pr(s|\omega=0)\Pr(V=0,M=1|m,\omega=0)}}$$

But since $q_{sm0} = q_{sm1}$,

$$\frac{\Pr(V = 0, M = 0|m, \omega = 1)}{\Pr(V = 0, M = 0|m, \omega = 0)} = \frac{\Pr(V = 0, M = 1|m, \omega = 1)}{\Pr(V = 0, M = 1|m, \omega = 0)}$$

and so both expressions are equal to

$$\frac{\Pr(V = 0|m, \omega = 1)}{\Pr(V = 0|m, \omega = 0)}$$

If $\Pr(V = 0, M = 1|m) = 0$, then $\Pr(V = 0, M = 0|m, \omega) = \Pr(V = 0|m, \omega)$ for any $\omega \in \{0, 1\}$. Therefore, for any $M \in \{0, 1\}$

$$q_{smM} = \frac{1}{1 + \frac{\Pr(s|\omega=1)\Pr(V=0|m,\omega=1)}{\Pr(s|\omega=0)\Pr(V=0|m,\omega=0)}}$$

The condition $q_{sm0} = q_{sm1}$ implies that each player's vote v does not depend on the opponent's message M, i.e. $\Pr(v = 0|M = 0, m, \omega) = \Pr(v = 0|M = 1, m, \omega)$. Since

$$\Pr(v = 0|M = 0, \omega) = \Pr(v = 0|M = 0, m, \omega)\Pr(m|\omega) + \Pr(v = 0|M = 0, m', \omega)\Pr(m'|\omega)$$
$$\Pr(v = 0|M = 1, \omega) = \Pr(v = 0|M = 1, m, \omega)\Pr(m|\omega) + \Pr(v = 0|M = 1, m', \omega)\Pr(m'|\omega)$$

it follows that $\Pr(v=0|M=0,\omega)=\Pr(v=0|M=1,\omega)$ and therefore

$$\frac{\Pr(v=0|M=1,\omega=1)}{\Pr(v=0|M=1,\omega=0)}=\frac{\Pr(v=0|M=0,\omega=1)}{\Pr(v=0|M=0,\omega=0)}=\frac{\Pr(v=0|\omega=1)}{\Pr(v=0|\omega=0)}$$

and, by symmetry,

$$\frac{\Pr(V=0|m=1,\omega=1)}{\Pr(V=0|m=1,\omega=0)}=\frac{\Pr(V=0|m=0,\omega=1)}{\Pr(V=0|m=0,\omega=0)}=\frac{\Pr(V=0|\omega=1)}{\Pr(V=0|\omega=0)}.$$

Therefore all the voting cutoffs coincide with

$$q_{smM}=\frac{1}{1+\frac{\Pr(s|\omega=1)\Pr(V=0|\omega=1)}{\Pr(s|\omega=0)\Pr(V=0|\omega=0)}} \tag{24}$$

This implies that any solution to the equation (24) also solves equation (22). Since the final decision does not depend on the messages, any symmetric non-responsive equilibrium is outcome-equivalent to a symmetric babbling equilibrium. ■

Proof of Proposition 3 (outcome-equivalence of class 0 and class 1 equilibria). The system that characterizes an equilibrium of class 0 is

$$\begin{cases} q_0^0=\dfrac{1}{1+R\frac{F(k_R(q_{010}^0)-F(k_R(q_{011}^0))+R[F(q_{010}^0)-F(q_{011}^0)]}{R[F(k_R(q_{010}^0)-F(k_R(q_{011}^0))]+F(q_{010}^0)-F(q_{011}^0)}} \\ q_{011}^0=\dfrac{1}{1+R\frac{[F(k_R(q_{011}^0)-F(k_R(q_0^0))+R(F(q_{011}^0)-F(q_0^0))]}{[(F(q_{011}^0)-F(q_0^0))+R\left(F(k_R(q_{011}^0)-F(k_R(q_0^0))\right)]}} \\ q_{010}^0=\dfrac{1}{1+R\frac{[F(k_R(q_0^0))+RF(q_0^0)]}{[F(q_0^0)+RF(k_R(q_0^0))]}} \end{cases} \tag{25}$$

The system for class 1 is

$$\begin{cases} q_{001}^1=\dfrac{1}{1+R\frac{(F(k_R(q_{010}^1)-F(k_R(q_0^1))+R(F(q_{010}^1)-F(q_0^1))}{R(F(k_R(q_{010}^1)-F(k_R(q_0^1))+(F(q_{010}^1)-F(q_0^1))}} \\ q_0^1=\dfrac{1}{1+R\frac{(F(k_R(q_0^1)-F(k_R(q_{001}^1))+R(F(q_0^1)-F(q_{001}^1))}{R(F(k_R(q_0^1)-F(k_R(q_{001}^1))+(F(q_0^1)-F(q_{001}^1))}} \\ q_{010}^1=\dfrac{1}{1+R\frac{F(k_R(q_{001}^1)+RF(q_{001}^1)}{RF(k_R(q_{001}^1)+F(q_{001}^1)}} \end{cases} \tag{26}$$

The two systems admit solutions (q_s^0, q_{smM}^0) and (q_s^1, q_{smM}^1) such that

$$q_s^0=q_{s01}^1,\quad q_{s11}^0=q_s^1,\quad q_{s10}^0=q_{s10}^1$$

■

Proof of Proposition 4. Suppose that both players have the same quality of information ($R_s=R_r=R$) and consider an equilibrium of the one-sender game in which $r_{s0} = q_{s1}$ and $r_{s1} = q_s$. The characterizing system is

$$\begin{cases} q_{01} = \frac{1}{1+R\frac{[RF(q_0)+F(k_R(q_0))]}{[F(q_0)+RF(k_R(q_0))]}} \\ q_0 = \frac{1}{1+R\frac{[R(F(q_{01})-F(q_0))+(F(k_R(q_{01}))-F(k_R(q_0)))]}{[(F(q_{01})-F(q_0))+R(F(k_R(q_{01}))-F(k_R(q_0)))]}} \end{cases}$$

The cutoffs (q_s^2, q_{smM}) for a class 2 equilibrium of the two-sender game are determined by

$$\begin{cases} q_{000} = \frac{1}{1+R\frac{[RF(q_0^2)+F(k_R(q_0^2))]}{[F(q_0^2)+RF(k_R(q_0^2))]}} \\ q_0^2 = \frac{1}{1+R\frac{[R(F(q_{000})-F(q_0^2))+(F(k_R(q_{000}))-F(k_R(q_0^2)))]}{[(F(q_{000})-F(q_0^2))+R(F(k_R(q_{000}))-F(k_R(q_0^2)))]}} \end{cases}$$

Given an equilibrium (q_s, q_{sm}) of the one-sender game, it is easy to show by inspecting the above equations that there exists a class 2 equilibrium (q_s^2, q_{smM}) such that

$$q_s = q_s^2, \quad q_{s1} = q_{s10}$$

The above equalities and the fact the the cutoffs q_{s11} and q_{s0} are irrelevant for the outcome imply that the two equilibria are outcome-equivalent. In the same way, given a class 2 equilibrium, it is possible to construct an outcome-equivalent equilibrium of the one-sender game. ■