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FINANCIAL CONSTRAINTS IN SEARCH EQUILIBRIUM[†]

Abstract

The Great Recession has indicated that firms' leverage and access to finance are important for hiring and firing decisions. It is now empirically established that bank lending is correlated to employment losses when credit conditions deteriorate. We provide further evidence of this drawing on a new dataset that we assembled on employment adjustment and financial positions of European firms. Yet, in the Diamond Mortensen Pissarides (DMP) model there is no role for finance. All projects that display positive net present values are realized and financial markets are assumed to be perfect. What if financial markets are not perfect? Does a different access to finance influence the firm's hiring and firing decisions? The paper uses the concept of limited pledgeability proposed by Holmstrom and Tirole to integrate financial imperfections and labor market imperfections. A negative shock wipes out the firm's physical capital and leads to job destruction unless internal cash was accumulated by firms. If firms hold liquid assets they may thus protect their em search capital, defined as the cost of attracting and hiring workers. The paper explores the trade off between size and precautionary cash holdings in both partial and general equilibrium. We find that if labor market frictions disappear, so does the motive for firms to hold liquidity. This suggests a fundamental complementarity between labor market frictions and holding of liquid assets by firms.

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Keywords: labor and finance, leverage and pledgeability

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1 Introduction

The 2008 financial crisis and the associated increase in unemployment on both sides of the Atlantic sparked a new interest in the relationship between financial imperfections and labor market dynamics. In the aftermath of the crisis, a growing empirical literature studied the links between financial conditions and employment adjustment. The Great Recession has indicated that firms' leverage and firms' access to finance are clearly correlated to hiring and firing decisions. More specifically, it is now empirically accepted that frictions in bank lending are correlated to employment losses when credit conditions deteriorate.[1]

The Diamond Mortensen Pissarides (DMP) model is the main paradigm for addressing imperfect labor markets. In the baseline framework, there is no role for finance. All projects that display positive net present values are realized and financial markets are assumed to be perfect. What if financial markets are not perfect? Does a different access to finance influence the firm hiring and firing decisions? These basic questions call for a deeper understanding of the relationship between labor and finance. Among the financial frictions addressed by the literature and reviewed below, this paper exploits the concept of limited pledgeability proposed by Holmstrom and Tirole (2011). The idea is that only part of the entrepreneur's income is pledgeable and can be borrowed upon, either because part of the income is private benefit or because the entrepreneur needs incentives. By adding financial imperfections and borrowing constraints into an otherwise standard equilibrium unemployment model, the paper contributes to the building of an archetype and flexible model of labor and finance.

In our model, firms are financially constrained by limited pledgeability and invest in physical capital within an imperfect labor market. Entering firms attract workers by posting vacancies with wages attached to them and hire up to an endogenously determined size level that depends on the firms' access to finance. Firms anticipate the possibility that new funding will be needed over the lifetime, and that refinancing may not be available in those times. If that happens, the firm must rely on internal funds for financing the rebuilding of its physical capital. In the absence of such funds, the firm is forced to fire workers and close down its operations. When workers are fired, the firm looses its *search capital*, defined as the cost of attracting and hiring workers. *Ex ante*, firms therefore face a trade-off between investing their limited funds in a *war chest* of liquid funds to protect their search capital, or to invest in more capacity (more employees). [2]

Our theoretical model shows that if labor market frictions disappear, so does the motive for firms to hold cash. This implies a fundamental complementarity between labor market frictions and holding of liquid assets by firms that is novel in the literature. In this sense, the paper brings together the work on liquidity by Holmstrom and Tirole (2011) with the traditional Mortensen Pissarides (1994 and 1999) model of equilibrium unemployment.

While we largely exploit the concept of limited pledgeability, other financial frictions have been proposed in the literature. In the early literature, Greenwald-Stiglitz (1993) looked at the risk aversion of firms. Farmer (1985) studied the financing of quasi-fixed costs, and Townsend (1979) proposed the costly verification model. Sticky bank borrower relationships also emerge in the context of asymmetric information with moral hazard (Holmostrom and Tirole, 1997) and adverse selection (Sharpe, 1990). Within a more labor oriented literature, Wasmer and Weil (2004) investigated the interplay between matching frictions in both the labor and the financial markets. Michelacci and Quadrini (2009) anal-

[1] Chodorow-Reich (2014) and Bentolila et al. (2014) use loan level data for the US and Spain during the 2007/09 financial crisis to identify the effects of banks health on employment changes; Boeri, Garibaldi and Moen (BGM, 2013) review the empirical literature and provide new evidence using macro, sectoral and firm-level data. Pagano and Piga (2010) use sectoral data to identify the impact of leverage and employment changes, using the methodology proposed by Rajan and Zingales (1998) to study the relationship between finance and growth.

[2] In an accompanying paper, Boeri Garibaldi and Moen (BGM, 2014) study the effects of limited pledgeability on job creation of new firms over the business cycle. Merz and Yashiv (2007) discuss the relationship between adjustment costs of labor and the value of the firm.

ysed the effects of financial market imperfections on employment adjustment and the size distribution of firms.

The structure of the paper is as follows. Section 2 presents some of the key empirical regularities between access to credit and employment changes using micro data from the Great Recession. Section 3 introduces the model, and characterizes the trade off between cash and finance in partial equilibrium. Section 4 derives the general equilibrium results. Section 5 discusses the key findings of our theory while section 6 concludes.

2 Some facts about access to finance and employment

This section presents some empirical regularities on the relationship between firm financial conditions and employment changes.

Specifically, we present facts based on on a dataset of firm-level employment adjustment and leverage during the Great Recession. The data cover the period 2007-9 and are obtained by matching data from the EFIGE survey of European firms with information from balance sheets obtained in the Amadeus archive. Efige samples some 16,000 European firms (3,000 in large countries, such as Germany, France, Italy, Spain and the UK, and 500 firms in smaller countries, such as Austria and Hungary). The data in the matched sample cover mainly large firms (the average firm size in terms of employees is 81).[3]

Our main variable of interest is employment changes.[4] In the appendix we plot the distribution of employment changes using a Kernel density estimator. As our data cover the Great Recession, most firms appear to be downsizing.

To summarize regularities and insights as as to the importance of finance in employment adjustment, Figure 1 plots the Kernel estimates for firms that successfully applied for credit (continuous line), as well as firms that did not apply for credit (dotted line) or that applied, but were not successful (dash line). The distribution of job losses among those that unsuccessfully applied for credit lies strictly above the other two distributions. This suggests that the firms that were un-successful in refinancing operations were, on average, heavily downsizing (on average by almost 20 %) while the distribution of employment adjustment among successful debtors and firms that did not apply for credit is remarkably similar (in the latter group there is only a larger proportion of firms not experiencing employment variations). The concentration of employment losses (about 30 per cent of the total) among firms experiencing difficulties in refinancing operations is obviously not informative as to causality: it may well be that firms did not obtain credit because they were downsizing and considered not be viable creditors by banks. Yet, the chart clearly reports a link between access to credit and employment changes, as we summarize in the following.

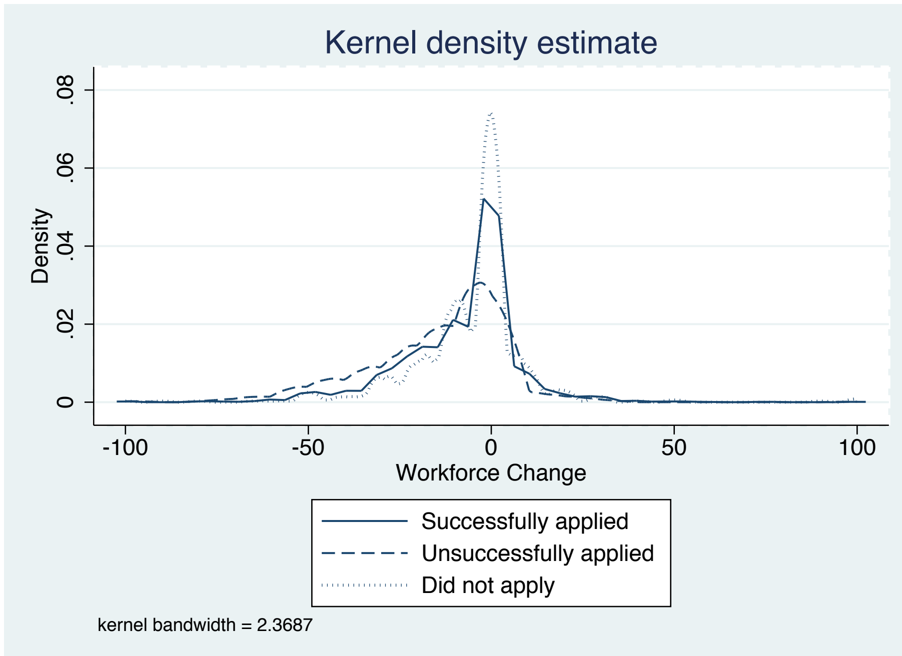
- **Fact 1** Job losses are negatively correlated with access to credit during the financial crisis

In addition to employment changes, we exploit measures of leverage in 2007, the year before the beginning of the Great Recession. In particular, the *Gearing ratio* is the debt to equity ratio measuring the extent to which the firm is using creditor's vs. owner's funds, whilst the *solvency* ratio measures the ratio of after tax net profit (excluding non-cash depreciation expenses) over debt and is a measure of one company's ability to meet long-term obligations. The appendix reports some descriptive statistics on those data.

[3]The questionnaire is very detailed on a number of structural characteristics of firms such as organization, job composition, innovation activities, finance as well as product and labor market strategies. The Amadeus archive provides financial and business data on Europe's biggest 500,000 companies by assets. Hence, the matched sample covers only the large firms and the cross-country comparability is limited

[4]We draw on the following question asked to employers at the beginning of 2010: *During the last year (2009) did you experience a reduction or an increase/decrease of your workforce in comparison with 2008?* . For those stating to have changed employment levels, a second question elicited the percentage change in the workforce. We imputed a zero value to firms declaring that they did not experience any change in employment in the first question

Figure 1: Firm-level net employment change, distribution of firms by access to credit



To correlate financial leverage to employment changes controlling for firm characteristics, we estimate a regression of changes in employment on firm, sector as well as aggregate country fixed effects, output variations as well as leverage. In particular, Table 1 reports estimates of the following equation

$$\Delta e_{ijc} = \alpha + \alpha_j + \alpha_c + \alpha_j * \alpha_c \beta \Delta y_{jc} + \gamma Lev_{ijc} + \delta S_{ijc} + \epsilon_{ijc} \tag{1}$$

where Δe is the reported employment growth rate *during* the period 2008-9, i denotes the firm, j the sector and c the country, S is set of size dummies (employment or turnover) and Lev is the Gearing Ratio, measured *before* the Great Recession (according to 2007 balance sheet data). Δy_{jc} is change in the sectoral output. We also include country and sector dummies as well as interactions between the two sets of dummies. We summarize these results in our second empirical regularity.

- **Fact 2** Financial leverage is negatively correlated to net employment changes during the crisis

Fact 2 is reported in Columns (1) and (2) in Table 1. The dependent variable is employment change. The gearing ratio is negatively associated with plant-level employment change, while the Solvency Ratio is positively associated with employment changes.

While these correlations are significant, leverage is clearly endogenous. The growing empirical literature that has used the Great Recession as an episode of credit contraction is concerned with the causal effect of credit contraction on employment. Chodorow-Reich (2014) for the U.S. and Bentolila et al. (2014) for Spain look at the health conditions of banks during the crisis as a way to identify the shock to credit independently of the firm conditions. They both found evidence of a *causal effect of credit disruption in employment losses.* We use our dataset to see whether we can confirm the following empirical regularity.

- **Fact 3** Financial leverage negatively affects employment changes during the crisis

Columns (3) to (6) of Table 1 display 2-stages least squares estimates in which leverage is instrumented by a dichotomic variable capturing firms that can use third party collateral being part of a consortium of firms. The underlying identification assumption is that the presence of this collateral affects the (equilibrium) level of leverage prevailing before the financial crisis while it does not directly affect employment variation during the Great Recession. The first-stage results point to a significant and positive (negative) effect of third party collateral on leverage (solvency). In the second stage we still find a negative and statistically significant effect of leverage and solvency on firm-level employment adjustment. The effects of leverage on employment adjustment is non-negligible: bringing, say, a typical Austrian firm to the average gearing ratio of a German firm involves additional employment losses of the order of 3 per cent during a financial recession; increasing by 10 basis points the solvency ratio (like moving an average Italian firm to France) involves a 6 per cent increase of employment. As shown by the bottom row of Table 8 , the 2SLS estimates have substantially less observations that the OLS estimates. This is because there are many missing values in the question about third party collateral. [5].

Where do these effects come from? Columns (5) and (6) display estimates of equation (1) when only firms downsizing or only firms up-sizing are considered. The focus is on leverage, but the results are the same when we consider solvency ratios. They suggest that after the financial crisis the effect of leverage on firm-level employment adjustment is driven by firms that are downsizing. For upsizing firms the second-stage coefficient is negatively, but not statistically signmificant[6]. Thus, we have

- **Fact 4** The effects of financial leverage on employment changes during a financial crisis are concentrated in downsizing firms.

Overall, our results suggest that leverage matters for employment adjustment during a financial recession and operates mainly along the gross job destruction margin. Ceteris paribus, more leveraged firms destroy more jobs than firms with a higher solvency ratio.

3 The model

Our starting point is a directed search model of the labor market, where entrepreneurs pay a fixed cost of entry and may potentially hire many workers. We then introduce financial frictions into the model. All agents in the model are risk neutral, and discount the future at the same rate r. Workers are infinitely lived.

Production technology

The entrepreneurs set up a firm at effort cost K. They then decide the size of the firm, or capacity, A (number of machines). The per unit investment cost is 1, hence total investment costs are A. The firm can not increase capacity at a later stage. The firm hires workers in a labor market with frictions. It takes one worker to operate one machine, hence the firm hires A workers, with a Leontief tecnhnology. The workers stay with the firm until their job is destroyed. Output is linear in the number of jobs with marginal productivity equal to y, so that $f(A) = yA$. The entrepreneurs receive an exogenous

[5] We did run regressions replacing missing values with 0, but did not find substantial differences

[6] We also run regressions including firm-level output growth (rather than the average growth rate at the sectoral level) as right-hand-side variable. Such a specification clearly creates a problem of endogeneity, but potentially captures idiosyncratic shocks unrelated to the financial recession. Also in this case, there is still an effect of leverage on employment growth. As a further robustness check we run regressions putting on the left-hand-side a categorical variable (0 for downsizing, 1 for firms keeping the same employment level, 2 for those upsizing) in order to cope with measurement error, notably heaping in the reporting of employment adjustment. There is still a statistically significant effect. Coefficients are remarkably stable across these different specifications

Table 1: Leverage and Employment Adjustment

	(1)	(2)	(3)	(4)	(5)	(6)
Variables	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$
Sample	All firms	All firms	All firms	All firms	Downsizing	Upsizing
Method	OLS	OLS	IV	IV	IV	IV
			Second stage			
$\Delta\bar{y}$	1.11	1.05	-57.31	98.56	95.87	33.53
	(0.910)	(0.901)	(133.3)	(169.1)	(132.8)	(679.9)
Gearing	-0.01***		-0.03**		-0.34*	-0.03
	(0.00)		(0.01)		(0.02)	(0.17)
Solvency		0.04***		0.60***		
		(0.01)		(0.21)		
Constant	-8.12***	-10.73***	-13.09	-13.19	-24.75	0.98
	(2.59)	(2.63)	(17.11)	(20.69)	(16.62)	(106.20)
Country	YES	YES	YES	YES	YES	YES
Sector	YES	YES	YES	YES	YES	YES
Country*Sector	YES	YES	YES	YES	YES	YES
Size	YES	YES	YES	YES	YES	YES
			First stage			
			Gearing	Solvency	Gearing	Gearing
Third party collateral			108.24***	-6.85***	88.37***	31.11***
			(16.48)	(1.69)	(21.31)	(68.12)
Observations	8596	9649	2358	2900	1195	307

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

income flow y_0, independently of production levels. Production is subject to adverse shocks. With a given probability rate λ, all the machines are destroyed, and the firm has to reinvest in order to continue production. Output after reinvestment is y times the investment level up to A. We refer to this event as a λ-shock. A second λ-shock kills the firm.

An important assumption is that a λ-shock terminates all contracts, both between the firm and its employees and between the firm and the bank. The assumption simplifies the analysis considerably. As will be clear below, it implies that the firm cannot borrow from its employees to get around the borrowing constraint it faces. Furthermore, it implies that the firm cannot use its income after the shock as a basis for loan from the bank. We may think that the loan from the bank is attached to the firm's capital, which serves as collateral, and that the firm cannot force the firm to repay the debt if the machines are destroyed.[7]

Search

The labor market contains frictions, and the frictions are modeled as in the Diamond-Mortensen-Pissarides framework. A constant return to scale matching function $x(u, v)$ maps stocks of searching workers u and firms with vacancies v into a flow x of new matches. In order to simplify some of the expressions we assume that the matching function is Cobb-Douglas, i.e., that $x(u, v) = u^{\beta} v^{1-\beta}$. Let $p(\theta)$ denote the job finding rate of searching workers and $q(\theta)$ the arrival rate of workers to searching firms, where $\theta = v/u$ is labor market tightness. Search is directed, and we use the competitive search equilibrium concept (Moen (1997), Shimer (1996), and Mortensen and Wright (2002))[8]. Let U denote the expected net present value of future income for an unemployed worker. It follows that

$$rU = z + \theta q(\theta) R \tag{2}$$

where R is worker rents associated with finding the job.[9]. In equilibrium, workers receive the same expected income independently of which firms they search for. Hence (2) defines a relationship between θ and the rents R offered by the firm. The firm can post as many vacancies it wants at cost c per vacancy. The probability rate of finding a worker when v vacancies are posted is vq, and the expected time to fill the vacancy is $1/vq$. In what follows we assume that the firm posts (infinitely) many vacancies. Hence it receives a worker immediately at search cost c/q. The firm thus faces a trade-off between wage costs (high R) and search costs c/q. As will be clear below, the firm chooses R so as to minimize what we refer to as the total hiring cost $C = c/q + R$. Hence C is given by

$$C = \min \left[\frac{c}{q(\theta)} + R \right] \qquad \text{S.T.} \qquad rU = z + \theta q(\theta) R \tag{3}$$

In the appendix we show that total hiring cost is

$$C = \frac{c}{q} \frac{1}{1-\beta}$$

where

$$\theta(U) = \frac{rU - z}{c} \frac{1-\beta}{\beta}$$

It follows that we can write both the total labor costs C and market tightness θ as an increasing function of U, $C = C(U); \theta = \theta(U)$ with $C'(U) > 0$ and $\theta'(U) > 0$. With Cobb-Douglas matching function, one can show that $C''(U) < 0$.

[7] If the bank sizes the machines, the firm cannot continue operating the firm, and the entrepreneur looses her nonpledgeable income. This disciplines her from repudiating on the debt. When the machines are gone, the bank has no assets to size, and the entrepreneur will repudiate on the loan.

[8] For a large-firm application (as in the present paper) see Kaas and Kircher(2013).

[9] If W denotes the NPV income of an employed worker, $R = W - U$

As stated above, all contracts, including labor contracts, are terminated when a λ-shock occurs. If the worker is retained after such a shock, the firm and the worker renegotiate the wage contract. Our assumption is thus that in this bargaining game, the firm has all the bargaining power, and the worker receives a wage of rU (the flow value of his outside option). Hence the firm cannot defer payment of worker rents (wages over and above rU) until after the machine breaks down. The wage profile up to the point where the shock hits is irrelevant. If the firm offers a constant wage, the relationship between this wage and R is given by

$$w = rU + (r+\lambda)R$$

Financial contracts

As stated initially, an entrepreneur buys machines and hires workers before production takes place. However, at this stage entrepreneurs have no funds, and hence have to borrow the necessary funds in order to invest.

We impose two financial frictions. The first concerns limited borrowing at the initial stage. The second concerns the possibility that the firm cannot get access to refinancing after a λ-shock. At both points our approach is borrowed from Holmstrom and Tirole (2011), and as them we do not explicitly model financial intermediation. We will discuss the two frictions in turns.

We model limited borrowing at the initial stage by assuming that the entrepreneur cannot commit to repay her entire future income to a creditor. More specifically, we assume that the entrepreneur can commit to repay her exogenous income y_0. The private income is necessary in order to get any borrowing at all. In addition, the entrepreneur can promise to repay a part but not all the income the project is expected to generate. This may be for several reasons. Firstly, one may assume that part of the gain from running a business is a private, non-pecuniary benefit. This can not easily be transferred to the creditor. Second, in order to incentivize the entrepreneur to make the right decisions, taking properly care of the machines and so on, the entrepreneur must have a sufficiently large stake in the project. We refer to the part of the income that the entrepreneur can commit to repay as the entrepreneur's pledgeable income.

The entrepreneur cannot save her non-pledgeable income. The assumption is easily rationalized if the non-pledgeable income is private benefits. However, the assumption is made for convenience. As long as accumulated retained earnings is likely to be less than the reinvestment needed, accumulated savings will only influence the size of the war chest, not whether the firm will have one or not (due to the linear structure of the model, to be explored below).

We assume that the non-pledgeable income is proportional to the number of machines the entrepreneur controls, i.e., it is equal to xA, where x is a parameter.[10]

$$\tilde{p} = y_0 + (y - w - x)A \tag{4}$$

The NPV of the pledgeable income writes[11]

$$\begin{aligned}\tilde{P} &= Y_0 + A\frac{y-w-x}{r+\lambda} \\ &= Y_0 + A\frac{y-x-rU}{r+\lambda} - AR \end{aligned} \tag{5}$$

[10] In an earlier version of the paper we showed that all the results also go through if we instead write the non-pledgeable income as a fraction of output net of the opportunity cost of workers, $\rho(y - rU)$, where ρ is a constant.

[11] We assume that $y - rU - x > 0$. This will be necessarily true in equilibrium.

where $Y_0 = y_0/(r+\lambda)$, and R is the rent associated with employment. If the firm borrows $\tilde{P}$, it pays back all its pledgeable income until the machine is destroyed, in which case the contract is terminated.

The second financial frictions concerns refinancing. Also at this point we follow Holmstrom and Tirole, by assuming that there is an exogenous probability, τ, that the firm will not get refinancing. If so, the firm has to close down unless it has available resources on its own. If the firm closes down, its search capital is lost.

The firm can use its available resources to invest in machines and search, or hold liquid reserves (deposits) that yield an interest rate r and ensure that- conditional on the adverse shock λ - a size I is available for investment in physical capacity. The *ex ante* cost of having I units of funds available when a λ-shock occurs is $\widetilde{\lambda} I$ where $\widetilde{\lambda} = \frac{\lambda}{r+\lambda}$ [12]. We may also think of I simply as an unused credit line that the firm has available and which is not destroyed by the λ-shock. We will often refer to the investment I as the firm's war chest.

The financial constraint the firm faces can thus be written as $\tilde{P} \geq (c/q+1)A + \tilde{\lambda} I$, or

$$Y_0 + A\frac{y - rU - x}{r+\lambda} \geq (C+1)A + \tilde{\lambda} I \tag{6}$$

since $C = c/q + R$. We denote the left-hand side of the equation by (6) by P. Hence, the borrowing constraint reads

$$P \geq (C+1)A + \tilde{\lambda} I \tag{7}$$

It follows from that the firm will always choose to set the workers' wage so as to minimize the total search cost C. We refer to this as *decoupling* between the firm's wage policy and the financial friction it faces.

Proposition 1 *Decoupling between wages and finance: Financial frictions do not directly influence the firm's wage setting*

Wages (above rU) and search costs tap equally much of the available funds, and the firm therefore minimizes the sum of the two, independently of the choice of I and A. Even though the wage payment occurs later, it is subtracted one to one from the pledgeable income, hence it creates the same financial burden as upfront investments in search costs.

4 Partial equilibrium

In this section we first derive the net present values, or 'asset values", of firms in different states of the world. Then we study the financial decision of firms.

Asset values

Let $W_1(A;I)$ denote the joint revenue obtained by a firm of size A with liquid deposit I available for reinvestment when a λ-shock hits.[13] Let W_2 and W_d be the NPVs of the joint revenues after the refinancing shock with access to finance, and after the shock with no access to the bank, respectively. Then

[12] This can be easily seen as

$$\begin{aligned} r\widetilde{\lambda} I &= \lambda(I - \widetilde{\lambda} I) \\ \widetilde{\lambda} &= \frac{\lambda}{r+\lambda} \end{aligned}$$

[13] For notational simplicity we do not include the exogenous income flow y_0 to the entrepreneur in any of the asset value equations. This is a matter of definition and unproblematic as long as y_0 is treated consistently in all equations.

$$rW_1(I,A) \quad = \quad yA + \lambda[\tau(W_d(I) + (A-I)U) + (1-\tau)(W_2(A) - (A-I)) - W_1(I,A)]$$

The first term shows the income flow of the machines. The second term reflects the capital loss associated with a reinvestment shock that happens with probability rate λ, after which the machines are destroyed and have to be replaced, and the firm looses W_1. After the shock, the firm gets refinancing with probability $1-\tau$, and regains W_2, the net present value of joint income of a fully financed old firms less the cost of reinvesting $A-I$. As the firm invests the liquid assets I, the cost in this case is simply $A-I$. With probability τ, the firm does not get refinancing, in which case the NPV of the joint income is W_d plus the outside option of the fired workers and the cost is fully paid by the liquid deposit I. It follows that

$$\begin{aligned} rW_2(A) &= yA - \lambda(W_2(A) - AU) \\ rW_d(I) &= yI - \lambda(W_d - IU) \end{aligned}$$

Solving gives

$$\begin{aligned} W_d &= \frac{(y+\lambda U)I}{r+\lambda} \\ W_2 &= \frac{(y+\lambda U)A}{r+\lambda} \end{aligned}$$

As the joint income $W_1(A,I)$ and $W_2(A)$, require A workers, we can define the surplus from the match of an entrepreneur and A workers as[14]

$$S_1(A,I) = W_1(A,I) - AU$$

It follows that[15]

$$S_1 = \frac{y-rU}{r+\lambda}(1+\tilde{\lambda}(1-\tau))A + \frac{y-rU}{r+\lambda}\tilde{\lambda}\tau I - \tilde{\lambda}(1-\tau)(A-I) \tag{8}$$

The first term reflects the NPV surplus of the A machines when refinancing using the war chest is not included. The second term reflects the additional expected income that the firm receives from using the war chest after an adverse shock and with no external finance available. The net value of a newborn firm thus reads

$$V(A,I) = S_1(A,I) - I\widetilde{\lambda} - (1+C)A$$

Using the definition of the surplus, and rearranging, the value of profits reads

$$V(A,I) \quad = \quad \left[\frac{y-rU}{r+\lambda} - 1\right]\left(1+\widetilde{\lambda}(1-\tau)\right)A + \left[\frac{y-rU}{r+\lambda} - 1\right]\tau\widetilde{\lambda}I - CA \tag{9}$$

Equation 9 is one of the key equations of the model and deserves some comments. Firm profits are a linear combination of investment in capacity, A, and investment in liquidity, I. The term in square

[14]Note that as y_0, the independent income flow of the entrepreneur, is not subtracted when calculating the surplus. This is consistent, since it is not included in the joint income either.

[15]This stems from the fact that

$$\begin{aligned} rS_1 &= (y-rU)A + \lambda[\tau(W_d - IU) + (1-\tau)(W_2 - AU - (A-I)) - W_1] + U \\ &= (y-rU)A + \tilde{\lambda}\tau I(y-rU) + \tilde{\lambda}(1-\tau)A(y-rU) - \lambda(W_1 - U) - \lambda(1-\tau)(A-I) \end{aligned}$$

from which (8) follows directly.

brackets is the real net internal return expressed as a present discounted value of the flow surplus $y - rU$. Note that the investment in real capacity A has a weight equal to $1 + \widetilde{\lambda}(1 - \tau)$ as only with probability $1 - \tau$ the firm finds refinancing and production can continue. Conversely, the investment in liquidity $\widetilde{\lambda} I$ improves production opportunities with probability τ, when the firm does not find refinancing and invests the warchest. Finally, the firm has to commit an amount CA to search costs (including worker rents).

Financial decisions

The firms' financial decision solves

$$\begin{aligned} V(U) &= \max_{A,I} V(A, I) \\ &\text{s.t.} \quad I\widetilde{\lambda} + (1 + C)A - P \geq 0 \\ &\qquad\; 0 \leq I \leq A; \;\; A \geq 0; \;\; I \geq 0 \end{aligned} \tag{10}$$

where- given the nature of the war chest and the structure of the shock- the war chest itself can not be larger than the investment A, while they both need to be non-negative.

Solving for A, and assuming that the borrowing constraint binds, gives

$$\begin{aligned} A &= \frac{Y_0 - \tilde{\lambda} I}{1 + C - \frac{y - rU - x}{r + \lambda}} \\ &= k(U)(Y_0 - \tilde{\lambda} I) \end{aligned} \tag{11}$$

We refer to the left-hand-side of the first equation as P. We refer to $k(U)$ as the *investment multiplier*. It shows the maximum units of capacity the firm can finance per unit of exogenous income Y_0 the entrepreneur is in possession of (from now on the dependence of U is surpressed). It follows that

$$\frac{dI}{dA} = -\frac{1}{k\widetilde{\lambda}} \tag{12}$$

so that the borrowing constraint is just a negatively sloped line in a (I, A) space.

The objective function makes it clear that the firm's value is a weighted average of investing in capacity A and accumulating a war chest I. The maximization problem is linear in A and I with a linear constraint and bounds on the endogenous variables I and A. Given the linear structure of the model, the firm's financial problem generically has a corner solution. Either the firm will go for maximum size, or it will hold cash so that it can refinance all the machines. In a *no cash* equilibrium, all firms set $I = 0$. Conversely, in a cash equilibrium, firms set $I = A$. By substituting the borrowing constraint into the objective function and taking derivatives, we find that the firm will choose to hold cash if

$$\left[\frac{y - rU}{r + \lambda} - 1\right](1 + \widetilde{\lambda}(1 - \tau)) \leq C + \left[\frac{y - rU}{r + \lambda} - 1\right]\frac{\tau}{k} \tag{13}$$

The left-hand side shows the gain from hiring one more worker. The right-hand side shows the gain from having $1/k$ more units in the war chest, including the search cost savings of not expanding capacity today.

Let D denote the difference between the right- and the left-hand-side in (13). We say that a high value of a parameter favors cash D if increasing in the parameter around the bliss point $D = 0$. We say that a high value of the parameter favors size if D is decreasing in the parameter.

Lemma 1 *In partial equilibrium, for a given U, the following holds*

1. *A high probability of distress, τ, favors cash*
2. *A high value of the pledgeability parameter x (large financial frictions) favors cash*
3. *A high value of the search cost c favors cash*

The first statement provides formally the idea that the warchest acts as a sort of insurance against distress. An increase in τ implies that the firm is more likely to loose access to financial markets conditional on an adverse shock λ. As a consequence, cash is more likely. An increase in pledgeability increases the financial resources available to the firm and reduces the incentives to hold cash. To understand the third statement, note that an increase in c induces an increase in the cost C and a reduction in the multiplier k. Both effects tend to increase the right-hand-side and make cash more likely.

5 General equilibrium

In general equilibrium, firms enter the market up to the point were the value $V(U)$ of entering is equal to the cost K of entering. Hence we can define general equilibrium as follows[16]

Definition 1 *The general equilibrium of the model is a vector (A, I, U, C) that satisfies*

1. *Optimal search behavior by firms: C is the solution to* (3)
2. *The firms' choice of capacity A and cash holdings I solves (10)*
3. *Free Entry, $V(U) = K$*

Let $V^A(U)$ denote the NPV value of a firm that maximizes size and has no cash ($I = 0$). Similarly, let $V^I(U)$ denote the value of a firm that holds cash and sets $I = A$. Clearly, $V(U) = Max\left[V^A(U); V^I(U)\right]$. From the envelope theorem it follows directly that $V^I(U)$ and $V^A(U)$ are strictly decreasing in U. It is also straight-forward to show that $V(U) \equiv \max\{V^I(U), V^A(U)\}$ is continuous and strictly decreasing in U. Existence and uniqueness thereby follows more or less directly.

Proposition 2 *The general equilibrium exists if*

$$\frac{y - z}{r + \lambda} > K$$

Generically, the equilibrium is unique

Note that for any given U, the firms choose one of the corners $I = A$ or $I = U$ unless $V^A(U) = V^I(U)$. The measure of the set of parameters that gives rise to this, using the n-dimensional Euclidean measure where n is the number of parameters, is zero. (In this case, the choice of A and I are indeterminate). Suppose therefore that the model exhibits multiple equilibria with different values of U. This cannot be the case, as both $V^A(U)$ and $V^I(U)$ and hence also $V(U) = Max\left[V^A(U); V^I(U)\right]$ are all strictly decreasing in U.

Corollary 1 *Suppose that unemployment income U in the two equilibrium candidates is U^A and U^I, respectively. Then the no-cash candidate is an equilibrium if and only if $U^A \geq U^I$, while the cash candidate is an equilibrium if and only if $U^A \leq U^I$*

[16]We do not specify unemployment rates and employment in new and old firms. See the appendix for details on labor stocks

Cash or size in general equilibrium

Parallel with our definition in partial equilibrium, we say that an increase in a parameter z (where z can be any parameter in the model) favors cash in general equilibrium if, from an initial situation where firms are indifferent between holding cash or not ($U^A = U^I$), an increase in z implies that all firms hold cash.

It is not trivial to see how parameters change the cash-size trade off, as shifts in parameters typically have several countervailing effects. In particular, studying the effects of parameter changes on the inequality (13) is a difficult route, as partial and general equilibrium effects tend to go in opposite directions.

Note, however, that at the point where firms are indifferent between holding cash and holding no cash, $V^I = V^A = V = K$. In particular, the zero profit condition for no-cash firms reads (from (9) and (11))

$$Y_0 k \left[\frac{y - rU}{r + \lambda}(1 + \tilde{\lambda}(1-\tau)) - (1 + C(U)) \right] \equiv K \tag{14}$$

Insert (11) into the expression for the value of a firm, (9), to get

$$V(I) = (Y_0 - \tilde{\lambda} I) k \left[\frac{y - rU}{r + \lambda}(1 + \tilde{\lambda}(1-\tau)) - (1 + C(U)) \right] + \tau \tilde{\lambda} I (\frac{y - rU}{r + \lambda} - 1)$$

Taking derivatives gives

$$V'(I) = -\tilde{\lambda} k \left[\frac{y - rU}{r + \lambda}(1 + \tilde{\lambda}(1-\tau)) - (1 + C(U)) \right] + \tau \tilde{\lambda} (\frac{y - rU}{r + \lambda} - 1)$$

Inserting from (14) gives

$$V'(I) = -\tilde{\lambda} \frac{K}{Y_0} + \tau \tilde{\lambda} (\frac{y - rU^*}{r + \lambda} - 1) \tag{15}$$

where U^* denotes the equilibrium value of U. The next lemma follows immediately

Lemma 2 *An increase in a parameter z favors cash if and only if it increases the righ-hand-side of (15).*

The lemma is very convenient in order to establish how the demand for cash is linked to aggregate variables. The following proposition follows almost immediately:

Proposition 3 *The following two results hold*

- *Increased search costs c favors cash, and in a frictionless market with $c = 0$, firms do not hold cash.*
- *An increase in y and in τ both favor cash. An increase in x and in unemployment benefits favors size.*

The proposition follows more or less directly from lemma 2. A formal proof is given in the appendix.

The first bullet point states that there is a complementarity between financial frictions and labor market frictions. In the presence of financial frictions, a firm's desire to hold cash is created by search frictions. Without search frictions, there is no search capital to protect, and the firm will not hold cash. Furthermore, as higher search frictions increase the search capital, increased search frictions make cash more likely.

Higher output means a tighter labor market, and this increases the value of the search capital. Hence, under higher productivity firms have stronger incentives to protect the search capital by holding cash.

Recall that τ reflects how frequently a firm cannot get refinancing, and hence can be considered as a measure of the quality of the financial system, with a low value of τ reflecting a high-quality financial system. The more likely it is that the financial system will fail, the stronger are the incentives to hold cash. Also the parameter x reflects the quality of the financial system. A higher x increases the shadow cost of holding cash. Again, a higher quality of the financial system favors size, and self-financing through cash becomes less attractive.

For changes in K and Y_0, proposition 3 (or lemma 2) gives us no guidance. The direct and indirect effects (through U^*) in (15) have different signs. Hence we are unable to derive general results on whether changes in K or Y favor cash or size.

Comparative statics within regimes

We can easily derive various comparative static results summarized in the following proposition.

Proposition 4 *In equilibrium, the following holds*

1. *A marginal increase in the difficulty of obtaining refinancing (an increase in τ), has no effect on the cash equilibrium, while it reduces welfare U in the no-cash equilibrium*
2. *Increased pledgeability (reduced x) increases the value of unemployment and the market tightness and reduces equilibrium unemployment in both types of equilibria.*
3. *An increase in firm productivity (y) increases the value of unemployment, market tightness and reduces equilibrium unemployment in both types of equilibria.*
4. *An increase in the entry cost, K, reduces the value of unemployment, market tightness and increases equilibrium unemployment in both types of equilibria.*

The proofs are omitted.

6 Extensions

Heterogeneous firms

In our framework all firms are identical, and hence face the same trade-off regarding cash vs size. In order to get cross-sectional differences, we have to introduce firms heterogeneity. To this end, suppose τ varies between firms[17]

Lemma 3 *Suppose that the conditions of proposition 2 for the existence of the general equilibrium are satisfied. Suppose further that the firms prefer cash if $\tau = 1$. Then there exists a unique τ^* ($0 \leq \tau^* \leq 1$) so that firms have cash if and only if $\tau > \tau^*$*

In the no-cash equilibrium candidate, U is strictly decreasing in τ. In a cash equilibrium candidate, U is independent of τ. Hence the crossing point is unique.

In light of the previous claim, we extend the model to allow firms to have different values of τ. All firms with τ on the same side of τ^* behave identically regarding whether or not to hold cash, and we therefore restrict ourselves to allow for two values of τ. More specifically, we assume that $\tau \in \{\tau^l, \tau^h\}$. Furthermore, we assume that the parameters are such that $\tau^h > \tau^* > \tau^l$. The firm specific value

[17]Note that we could just as well impose heterogeneity in terms of x rather than τ

of τ is determined upon entry, after K is sunk, but before the investment decisions are made. With probability δ, $\tau = \tau^l$, and with the complementary probability, $\tau = \tau^h$. The equilibrium value of U is determined, so that firms break even in expected terms.

The firms with a low τ will be more leveraged than firms with high τ. Hence, the model implies that leveraged firms are larger, more exposed to refinancing risk, and fire more workers when refinancing fails than do firms with a high value of τ.

Financial crisis

We define a financial crisis as a situation in which a subset of the banks require that firms repay an amount $T < P$ immediately. We may think of this as credit facilities (credit lines) suddenly drying up. We assume that the crisis only lasts for an instance, so that U is not affected. Finally, we assume that the crisis is unanticipated.

For firms with cash, the forced repayment shock does not create problems, as they can use their war chest to repay T. Furthermore, since the shock is for an instant, the probability that a refinancing shock occurs during the crisis is zero.

In order for a firm without cash to repay, it has to sell off its machines. Suppose that the scrap value of a machine is κ, $\kappa > \frac{y-rU-x}{r+\lambda}$ (i.e. the firm can still pay off its debt). In order to repay the loan, the firm sells machines and lay off workers. It will have to lay off a total of H/κ workers.

Proposition 5 *Suppose that a financial crisis hits, in the form of a repayment shock H. This has no effect on firms with cash holdings. Firms without cash holdings fire H/κ workers, and the unemployment rate increases.*

If the firm has to pay a firing cost to the replaced workers, this will increase the amount of firing the firm has to undertake. If the firing tax is t, the firm has to fire a total of $H/(\kappa - t)$ workers. However, as we have not included firing costs from the outset, we did not include this in the proposition.

After the financial crisis, the firm may start hiring again. However, resources are lost. When funds again are available, the amount of pledgeable income the firm can borrow is smaller than before the crisis, hence the firm cannot scale up the loan to the pre-crisis level. To be more specific, for each machine that is sold, the borrowing potential that is unlocked is $\kappa - X$. The income the firm can borrow on after the crisis is thus

$$\Delta Y = (\kappa - \frac{y - rU - x}{r + \lambda})\frac{H}{\kappa} \tag{16}$$

where $X = \frac{x}{r+\lambda}$. Let a denote the share of the search cost C the firm has to incur to obtain workers. The expansion in employment in the aftermath of the crisis is thus

$$\Delta A = \frac{\kappa - \frac{y-rU-x}{r+\lambda}}{1 + aC - \frac{y-rU-x}{r+x+\lambda}}\frac{H}{\kappa} \leq \frac{H}{\kappa} \tag{17}$$

The number of new hires is equal to the number of fired workers if $a = 0$ and $\kappa = 1$, i.e., if the scrap value of the machine is the same as the value of a new machine and the fired workers can be rehired at no costs. In all other cases, less workers are re-hired.

Proposition 6 *A repayment shock as described above permanently reduces the employment level in the firms involved.*

7 Discussion

In this paper we integrate limited pledgeability with labor market imperfections. We construct an archetype model for analysing the interplay between labor and financial imperfections. In addition, there are three main lessons that can be learned from our theory.

First, we uncover a key complementarity between firms holding cash and labor market imperfections. In our model the corporate sector holds cash as a way to protect its search capital. The latter is defined as the total hiring cost associated to labor market imperfections. The model predicts also that firms do not hold cash when labor market frictions disappear. While we are aware that the precautionary motives for firms holding cash are many, the complementarity between liquid assets and labor market imperfections is novel and should be investigated in future empirical work. [18] If we take literally the structure of our model, the larger are the labor market frictions, the larger should be the amount of cash held within the firms. We thus expect that firms operating within very tight labor markets will be more proned to hold liquid assets. Future empirical research may assert this relationship in detail.

Second, our theory predicts that those firms that hold more cash should be more protected to adverse shock hitting their lender. The recent empirical evidence, as well as the facts and regressions reported in section II, suggest that more leveraged firms dismissed more workers during the Great Recession. In our model, the firm borrows $\tilde{P}$ given by (5). The value of the firm is the entrepreneurs investments $K + Y_0$. Leverage is loans divided by total assets, and is given by

$$LE = \frac{\tilde{P}}{\tilde{P} + K + Y_0} \tag{18}$$

Since $\tilde{P}$ is increasing in A, leverage is higher in the no-cash than in the cash equilibrium. Furthermore, firms with $\tau < \tau^*$ should be less leveraged than firms with $\tau > \tau^*$. Our model predicts that there is more firing in the no-cash equilibrium during the crisis, and that no-cash firms fire more than firms with cash. This is consistent with our motivating facts.

Third, our theory predicts that firms embedded into better functioning financial sectors should be on average more leveraged and less inclined to hold cash. In addition, the theory predicts that a more financially integrated system should dismiss more labor under adverse financial conditions. We believe that the dynamics of US labor market in early 2007, when compared to the European experience in the aftermath of the financial shock, is revealing in this respect. The US corporate sector is arguably more financially integrated than the European one (Rajan Zingales, 1998). When the financial shocks hit in 2007, the US unemployment rose quickly from 5 to 9 percent, while European unemployment rose only modestly. It is certainly true that labor market institutions in Europe reduced labor shedding, but the dramatic rise in US unemployment is likely to have been the counter part of its finance orientation. The evidence reported in Boeri Garibaldi and Moen (2013) is coherent with this interpretation.

Admittedly, there is a caveat to the last argument. Although recent micro evidence assembled for the US (Chodorow-Reich, 2014) clearly suggests that the health conditions of the lender had a significant impact on the firm propensity to reduce employment during the financial crisis, this evidence is silent on the role of cash holdings. Indeed, the large firms in the US corporate sector became a net lender at the beginning of the 2000s. Armenter, Hnatkovska, (2012) show that in a sample of 6000 listed firms in Compustat, 44 percent had positive net financial asset in 2007, at the outset of the financial crisis.[19] How is it possible to reconcile the importance of cash holdings in

[18] Opler et al. (1999) argue that in general there are precautionary and transaction motives for the firms holding cash. First, the firm saves transaction costs to raise funds and does not have to liquidate assets to make payments. Second, the firm can use the liquid asset to finance its day to day activities if other sources of funding are not available. Armenter and Hnatkovska (2012) argue that firms accumulate cash holdings in order to avoid being financially constrained in the future. In their paper firms operate within a perfect labor market and must resort to costly equity every now and then. It turns out that the value function is strictly concave even if their utility is linear.

[19] Kararbounis and Meinman, 2012 link changes increase in corporate savings to changes in labor income shares

the US with the large employment losses observed in 2007 and linked to the lender health by the Chodorow-Reich (2014) ? We will argue that this does not count against our model, for at least three reasons. First, our model is best suited to describe small, privately held firms, while the liquid assets were held mainly by large listed (often multinational) firms. Second, and most importantly, Huasheng et al. (2012) clearly show that private firms hold less than half as much cash as public firms do. In addition, they also reports evidence the private firms adjust more slowly to their desired cash holdings. Lastly, the listed firms in Compustat held liquid funds in 2007 according to the accounting classification, but such liquid funds were not necessarily invested in cash. They may indeed have been very illiquid, particularly when the crisis hit.

8 Final remarks

We develop a micro-founded model of labor-finance interactions generating endogenously a demand for liquid assets. The models yields a number of testable implications. The most relevant in the context of the Great Recession is that highly leveraged firms should experience larger employment losses during a financial crisis. Recent empirical evidence reviewed in the paper as well as micro data on employment adjustment and balance sheets indicate indeed that highly leveraged firms and sectors are characterized by higher job destruction rates during financial recessions.

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Theoretical annex

Derivation of optimal search equations

The constraint implicitly defines an indifference curve $\theta = \theta(R, U)$ where U is the given value of unemployment. Further

$$\frac{d\theta}{dR} = -\frac{\theta q(\theta)}{q(1-\beta)R}$$

where β is the absolute value of the elasticity of $q(\theta)$, independent of θ under a Cobb-Douglas specification of the matching function. Total search cost define implicitly an isocost and the equilibrium is going to be a tangency condition between the isocost C and the indifference curve U.

Formally, the first order condition for a minimum- once we use the indifference curve is thus

$$\frac{cq'(\theta)}{q^2}\frac{\theta q(\theta)}{q(1-\beta)R} = 1$$

or

$$R = \frac{c}{q}\frac{\beta}{1-\beta}$$

Total labor cost is thus

$$C = \frac{c}{q}\frac{1}{1-\beta}$$

Over and beyond the rent, the firm pays the worker a flow value rU per period employed, as we further discuss at the end of this section. Finally, θ is given by

$$\theta q(\theta) = \frac{rU - z}{R} = (rU - z)\frac{1-\beta}{\beta}\frac{q}{c}$$

hence

$$\theta(U) = \frac{rU - z}{c}\frac{1-\beta}{\beta} \tag{19}$$

Worker flows and stocks

To complete the specification of the economy we have to account for the aggregate labor flows. In the economy there is a measure 1 of workers who can be employed in new firms or firms that already experienced the first λ shock. We label respectively n_1 and n_2 the share of workers employed in the two types of firms. In the war chest equilibrium, conditional on a λ shock firms do not fire any worker and continue with their cash holdings. Let ω be an indicator function that takes value 1 if the economy is in a no-cash equilibrium. The general balance flow conditions read

$$\begin{aligned}
\theta q(\theta)u &= \omega\lambda n_2 + (1-\omega)(\lambda\tau n_1 + \lambda)n_2 \\
\omega\lambda n_1 + (1-\omega)(\lambda(1-\tau))n_1 &= \lambda n_2 \\
u + n_1 + n_2 &= 1
\end{aligned}$$

The first equation is simply the outflows from unemployment and inflows into unemployment, where the latter involve also the share of workers in type 1 firms that do not find refinancing in the no cash equilibrium. The second condition is the flow into n_2 from type 1 firms and outflows out of n_2. Again, in the no cash equilibrium only the surviving employed enter the type 2 state. The last condition is the aggregate labor market condition. Solving for the stock yields

$$\begin{aligned}
u &= \omega\frac{\lambda}{\lambda + 2\theta q(\theta)} + (1-\omega)\frac{\lambda}{\lambda + (1+(1-\tau))\theta q(\theta)} \\
n_1 &= \omega\frac{\theta q(\theta)}{\lambda + 2\theta q(\theta)} + (1-\omega)\frac{\theta q(\theta)}{\lambda + (1+(1-\tau))\theta q(\theta)} \\
n_2 &= \omega\frac{\theta q(\theta)}{\lambda + 2\theta q(\theta)} + (1-\omega)\frac{\theta q(\theta)(1-\tau}{\lambda + (1+(1-\tau))\theta q(\theta)}
\end{aligned} \tag{20}$$

Proof of corollary 1

Proof Consider the two equilibrium candidates $V^A(U^A) = k$ and $V^I(U^I) = K$. Suppose $U^I > geqU^A$ then

$$K = V^A(U^A) > V^I(U^A)$$

but if $U^I > U^A$ then $V^I(U^a) > V^I(U^I)$ where the latter condition follows from the monotonicity of V^I. But the latter is a contradiction since it implies that U^I is simultaneously $V^I(U^I) = K$ and $V^I(U^I) < K$. So it must be that $U^A > U^I$. *QED.*

Proof of proposition 3

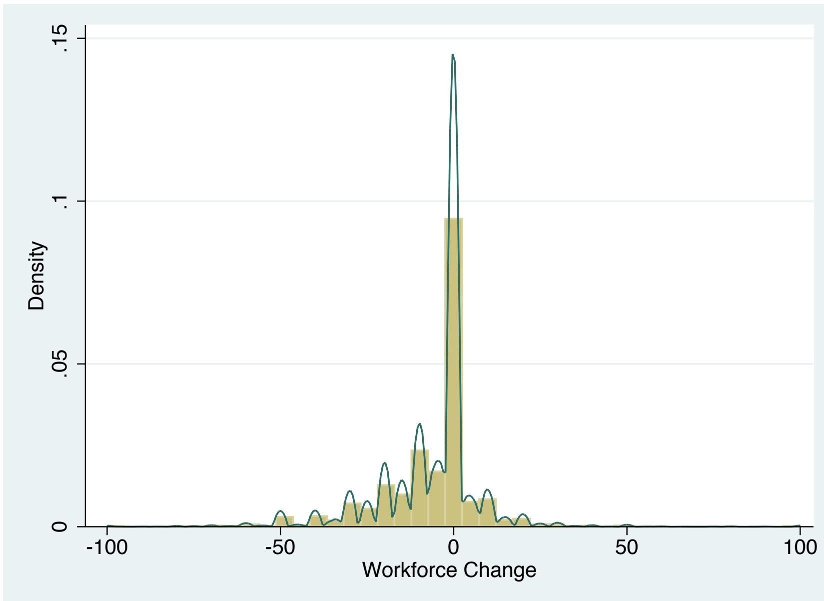
Proof: It is straightforward to show that U^* is decreasing in c. It follows that an increase in c increases the right-hand side of (15), and hence favors cash. Furthermore, in the limit, as $c \to 0$, one can easily show that $C \to 0$, $\theta \to 0$ and $R \to 0$. The labor market is competitive with a wage $w = rU < y$.[20] Equation (13) then reads

$$(1 + \widetilde{\lambda}(1-\tau)) \leq \frac{\tau}{k}$$

where $k = \frac{1}{1-\frac{y-w-x}{r+\lambda}} > 1$. As the left-hand-side is strictly greater than one, while the right-hand-side is strictly less than one, the result follows.

[20] Even in the limit, firms don't grow infinitely due to the borrowing constraint, hence wages must be below productivity in order for the firms to capitalize on K.

Figure 2: Firm-level net employment change, Distribution of firms



An increase in y increases $y - rU$. Suppose not. Then it follows from (9) that profits per worker fall strictly, and from (11) that the financial constraint tightens. Hence profits fall, a contradiction. It follows that $y - rU$ decreases, and hence that cash is more likely. An increase in τ reduces V^A while it does not influence V^I. An increase in τ therefore makes cash more likely. Finally, an increase in x increases U^*, and hence reduces the left-hand-side of (15).

Equilibrium when $c \to 0$

When $c \to 0$, it follows that $w \to rU < y$ and that $C = 0$ (the firms still have finite size and have to capitalize the investment K, hence $w < y$). It follows from (13) that the firm will hold cash whenever

$$\left[\frac{y-w}{r+\lambda} - 1\right](1 + \tilde{\lambda}(1-\tau)) \leq \tilde{\lambda}\tau \left[\frac{y-w}{r+\lambda} - 1\right]\frac{1}{k(w)\tilde{\lambda}} \tag{21}$$

This immediately simplifies to

$$k(w)(1 + \tilde{\lambda}(1-\tau)) \leq \tau \tag{22}$$

Since $k(w) \geq 1$ (it is 1 if the firm does not borrow from the bank), the inequality is always satisfied.

Statistical annex

Figure 2 plots the distribution of employment changes across firms in the EFIGE survey, using also a Kernel density estimator (blue line) to characterise the distribution. As data refer to a global recession year, most firms appear to be downsizing: the median is 0, the mean is -6. In addition to the mode at 0, there are also some spikes at -10, -20 and -30. This may indicate that respondents answered doing some rounding. Some of our estimates below take into account of such heaping problems.

Table 2: Measures of Leverage, Descriptive Statistics

Country	Number of Firms	Revenues growth 2008-2009		Size of Firms (Employees 2007)	
		Average	Standard Deviation	Average	Standard Deviation
AUT	443	63.5%	0.740	100	33
FRA	2,973	-8.3%	0.010	50	8
GER	2,935	-5.2%	0.008	96	11
HUN	488	-12.4%	0.015	68	9
ITA	3,021	-18.6%	0.005	40	2
SPA	2,832	-16.8%	0.015	45	3
UK	2,067	0.1%	0.032	20	773
Variable	Average	Min	Max	Standard Deviation	
Gearing Ratio (2007)	1.20	0.00	997.53	175.46	
Δe	-6.18	-100	100	15.16	
$\Delta \bar{y}$	-0.09	-0.30	5.64	0.39	
Size of Firms (2007)	116.65	0.00	365,630	3,595.00	
Third Party Collateral	0.04	0	1	0.21	

Table 2 provides some descriptive statistics on the measures of leverage which are used in the empirical analysis in 2007, the year before the beginning of the Great Recession. In particular, the *Gearing ratio* is the debt to equity ratio measuring the extent to which the firm is using creditor's vs. owner's funds. As shown by table 2, there is significant cross-country and within country (across sectors) variation in these measures. At the same time, there are large differences in the average size of firms across countries, which confirms that data are not cross-country comparable.

Table 3 reports OLS and IV regressions limited to either firms downsizing or expanding employment levels. In the 2SLS estimates the instrumented gearing ratio is significant only in the case downsizing firms.

The effects of leverage survive when we put on the left-hand-side a categorical variable (0 for downsizing, 1 for firms keeping the same employment level, 2 for those upsizing) in order to cope with the heaping problems mentioned above.

Table 3: Regression on downsizing and expandind firms

	Sample: Only Firms Downsizing	
	(3)	(4)
VARIABLES	$\Delta e(\%)$ Employment Growth	$\Delta e(\%)$ Employment Growth
Method	OLS	IV
	Second stage	
Gearing [a]	-0.003**	-0.034*
	(0.001)	(0.017)
$\Delta \bar{y}$ [b]	0.547	95.87
	(1.243)	(132.8)
	First stage	
		Gearing
Third party collateral		88.366***
		(21.310)
Observations	4151	1195

	Sample: Only Firms Upsizing	
	(5)	(6)
VARIABLES	$\Delta e(\%)$ Employment Growth	$\Delta e(\%)$ Employment Growth
Method	OLS	IV
	Second stage	
Gearing [a]	-0.0041*	0.0354
	(0.0023)	(0.171)
$\Delta \bar{y}$ [b]	-0.0915	33.53
	(3.048)	(679.9)
	First stage	
Third party collateral		31.11***
Observations	1060	307

All regressions include a constant and dummies for , Country, Sector, Size and Country*Sector

[a] Gearing Ratio is the debt to equity ratio

[b] Change in output at the sectoral level

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 4: All Firms

	(1)	(2)	(3)	(4)	(5)	(6)
VARIABLES	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$
Method	OLS	OLS	IV	IV	IV	IV
Second stage						
$\Delta\bar{y}$	1.107	1.049	-57.31	98.56	-95.87	-33.53
	(0.910)	(0.901)	(133.3)	(169.1)	(132.8)	(679.9)
Gearing	-0.004***		-0.029**		-0.34*	0.0354
	(0.001)		(0.012)		(0.017)	(0.171)
Solvency		0.04***		0.603***		
		(0.006)		(0.213)		
Constant	-8.123***	-10.73***	-13.09	-13.19	-24.75	0.976
	(2.594)	(2.630)	(17.11)	(20.69)	(16.62)	(106.2)
ountry	YES	YES	YES	YES	YES	YES
Sector	YES	YES	YES	YES	YES	YES
Country*Sector	YES	YES	YES	YES	YES	YES
Size	YES	YES	YES	YES	YES	YES
First stage						
			Gearing	Solvency	Gearing	Gearing
Third party collateral			108.24***	-6.846***	88.366***	31.11***
			(16.476)	(1.686)	(21.310)	(68.121)
Observations	8596	9649	2358	2900	1195	307

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Table 5: All firms (Δe categorical)

	(1)	(2)	(3)
VARIABLES	$\Delta e(\%)$	$\Delta e(\%)$	$\Delta e(\%)$
$\Delta\bar{y}$	-0.0157	-0.0165	-0.0201
	(0.0420)	(0.0411)	(0.0426)
Gearing	-0.000160***		
	(3.95e-05)		
Solvency		0.00104***	
		(0.000293)	
Constant	0.589***	0.522***	0.541***
	(0.120)	(0.120)	(0.119)
Country	YES	YES	YES
Sector	YES	YES	YES
Country*Sector	YES	YES	YES
Size	YES	YES	YES
Observations	8,693	9,757	8,161
R-squared	0.078	0.076	0.072

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1