Job to Job Movements in a Simple Search Model

By Pietro Garibaldi and Espen R. Moen*

On the job search is a key feature of real life labor markets. In this paper we present a tractable search model, closely related to the competitive model, in which on the job search and wage differentials for identical workers are an optimal response to search frictions and heterogeneous firms. Our model is laid out in detail in ongoing research by Pietro Garibaldi and Espen R. Moen (2009).

The model has three key elements. First, it applies the competitive search equilibrium concept, initially proposed by Moen (1997). Thus, firms post wages and vacancies to minimize search and waiting costs, and the labor market is endogenously separated into submarkets. Second, firms have convex costs of maintaining vacancies (in our simulations, the number of vacancies per firm is fixed). Third, contracting between a firm and its employees is efficient, so that their joint income is maximized.

The model tends toward an equilibrium characterization in which there is a job ladder in the labor market. Low productivity firms pay low wages, face high turnover rates, grow slowly and hire directly from the unemployment pool. More efficient firms pay higher wages, grow more quickly and hire from the employment pool. This characterization is qualitatively consistent with a variety of stylized facts about industry dynamics and worker flows: (i) workers move from low wage to high wage occupations, (ii) more productive firms are larger and pay higher wages than less productive firms, (iii) job to job mobility falls with average firm size and worker tenure, (iv) wages increase with firm size, and (v) wages are higher in fast-growing firms.

We also show that compared to traditional labor market models, our equilibrium model with on the job search delivers unexpected effects, even though it converges to traditional models as a special case (Chris A. Pissarides 2000). We argue that an increase in average productivity, caused by an exogenous shift in the fraction of high type firms in the market, can actually lead to an increase in unemployment and a reduction in entry for a subset of the parameter space. Complex, albeit intuitive, composition effects between queue length across different submarkets rationalize these findings.

Pissarides’ (1994) seminal paper on on the job search utilizes Diamond-Mortensen-Pissarides type of matching models. The most used model of on the job search in empirical research is Kenneth Burdett and Dale T. Mortensen (1998) and its follow-ups, where firms post wages and there is no matching function. Moen and Åsa Rosen (2004) were the first to analyze competitive and efficient on the job search. Guido Menzio and Shouyong Shi (2008), Rasmus Lentz and Mortensen (2007), and Giuseppe Moscarini and Fabien Postel-Vinay (2009) are currently studying models of on the job search.

The paper proceeds as follows. Section I introduces the structure of the model and characterizes the equilibrium. Section II shows the nonstandard effects of average productivity with a simple set of simulations.

I. The Model and Equilibrium

The labor market is populated by a measure one of identical workers. Individuals are risk neutral, infinitely lived, and discount the future at rate $r$. The technology requires an entry cost equal to $K$. Conditional upon entry, a firm learns its productivity and operates with a constant returns to scale technology. The productivity can take two values: a low value $y_1$ or a high value $y_2$, with probabilities $1 - \alpha$ and $\alpha$, respectively. The productivity of a firm is fixed throughout its life. Unemployed workers have access to an income flow $y_0 < y_1$. Firms exit the market at a constant, time independent rate $\delta$.

Firms decide how many vacancies to post and what (net present value of) wages to attach to
them. Each firm attaches the same wage to all its vacancies, but different firms of the same type may post different wages. The maintenance cost of vacancies is given by an increasing and convex function $c(v)$ with $c'(0) = c(0)$. Workers exogenously leave the firm at rate $s$.

Search is directed. Firms post vacancies and wages to maximize expected profits. They face a relationship between the wage they set and the arrival rate of workers to vacancies, which is derived from the indifference constraint of workers. Given this relationship, firms set wages so as to maximize profits.

As anticipated, we assume throughout that the firms and workers contract efficiently. In other words, the wage contract maximizes the joint income of the worker-firm pair. This simple assumption implies that a worker’s on the job search behavior internalizes fully the loss of profit incurred by the firm when she finds a new job. There are various wage contracts that may implement this behavior. For example, the worker pays the firm its entire net present value (NPV) up front and then gets a wage equal to $y_1$. In other words, the worker buys the job from the firm and acts thereafter as a residual claimant. Alternatively, the worker gets a constant wage and pays a quit fee equal to the firm’s loss of profit if a new job is accepted (see Moen and Rosen 2004 for more examples). In any event, the wage paid to the worker in the current job does not influence her on the job search behavior.$^1$

A submarket is characterized by an aggregate matching function bringing together the searching workers and the vacant firms in that submarket. In equilibrium, up to three submarkets may be operating: unemployed workers searching for low type jobs (the 01 market), workers employed in low type firms searching for a job in a high type firm (the 12 market), and unemployed workers searching for high type jobs (the 02 market). As explained below, the first two submarkets are always active (attract agents), while the 02 market may or may not be active, depending on the parameter values. In all submarkets the matching technology is the same. Suppose a measure of $N_{ij}$ workers search for a measure of $V_{ij}$ vacancies. We assume a Cobb-Douglas matching function $x(N_{ij}, V_{ij}) = AN_{ij}^{\alpha}V_{ij}^{1-\beta}$. The transition rates for workers and for firms are

$$p_{ij} = A\theta_{ij}^{1-\beta}$$

$$q_{ij} = A\theta_{ij}^{\beta}$$

where $\theta_{ij} = V_{ij}/N_{ij}$ is the labor market tightness in the market. Inverting the first of the previous conditions provides $\theta_{ij} = A^{-1/(1-\beta)}p_{ij}^{1/(1-\beta)}$ so that the transition rate for vacancies can be expressed as

$$q_{ij} = A^{1-\beta}p_{ij}^{-\beta}.$$

A. Worker Search

Let $M_i$ be the expected joint income of a worker hired in a firm of type $i$.$^2$ Then

$$rM_i = y_i + (s + \delta)(M_0 - M_i)$$

$$+ \max_j p_{ij}[W_j - M_i].$$

The first term is the flow production value created on the job. In addition, the current job can be destroyed for exogenous reasons at rate $s + \delta$. In this case the worker becomes unemployed and receives $M_0$ while the firm gets zero (for unemployed workers, the term is zero). Finally, the probability that the worker finds a new job is $p_{ij}$. In this event, the worker receives a NPV wage $W_j$ while $M_i$ is lost. Efficient on the job search implies that the workers search so as to maximize $M_i$. Since the wage paid by the firm is a pure transfer to the worker, it does not appear in the expression.

Workers employed in type 2 firms don’t search, hence

$$M_2 = \frac{y_2}{r + s + \delta}. $$

For searching workers (unemployed workers and workers employed in low type firms) the indifference curve $p_{ij}^*(W; M_i)$ shows combinations of $p$ and $W$ that provide the worker with $NPV$ income of $M_i$. It follows that

$^1$ It follows that a worker in a low type firm will never search for a job in other low type firms, as these cannot offer a wage that exceeds the productivity in the current firm.

$^2$ We have simplified the model layout by collapsing the asset value equations for unemployed and employed workers. If $i = 0$, the worker is unemployed, and the “joint income” is the income of the worker.
for $i = 0, 1$. Garibaldi and Moen (2009) show that the indifference curves cross only once, say $\pi_i (M_j) = \pi_j (M_j)$, at $W = W'$. For wages below $W'$, $p^w_0, p^w_1$. Hence, if a firm advertises a wage below $W'$, unemployed workers accept a lower job finding rate (a lower labor market tightness) than employed workers, and the firm attracts only unemployed applicants. If the firm advertises a wage above $W'$, the opposite happens, and the firm attracts employed workers only. In this way workers self-select into submarkets.

**B. Firm Search and Wages**

Firms decide on the number of vacancies to post and the wages attached to them. This influences profits only through future hirings and is independent of the stock of existing workers. At any point in time, a firm maximizes the flow value of search, given by $\pi = c(v) + vq[M_j - W]$

Suppose a firm of type $j$ decides to search for a worker of type $i$. Its maximization problem then reads:

$$\max_{W,v} -c(v) + vq[M_j - W]$$

subject to

$$q = q(p^w_{ij}(W, M_j)).$$

The resulting values of $p, W$, and $\pi$ define $p_{ij}, W_j$, and $\pi_{ij}$. The first order conditions read

$$W_j = M_j + (M_j - M_i)\beta$$

$$c'(v) = (1 - \beta)(M_j - W_j)q(p)$$

Using (2) gives

$$M_i = y_i + \beta p_y(M_j - M_i) \quad (5)$$

$$c'(v) = (1 - \beta)(M_j - W_q)p_{ij}q(p) \quad (6)$$

Finally, define $\pi_j = \max \pi_{ij}$.

Since $y_1 > y_0$ and workers search equally efficiently on and off the job, the submarket $0_1$ will attract both workers and firms. Furthermore, the $1_2$ market will also always be open. If not, a high type firm that opens vacancies with a wage slightly above $y_1$ would attract applications for all workers employed in type 1 firms. Hence $g$ and thus also profits would be infinite, which is inconsistent with equilibrium. The $0_2$ market may or may not be open depending on parameter values.

Finally, the expected profits of a firm of type $j$ entering the market can be written as

$$\Pi_j = \frac{\pi_j}{r + \delta}. \quad (7)$$

**C. Equilibrium**

Let $N_i$ denote the measure of workers in type $i$ firms. The aggregate resource constraint naturally implies that $\sum^n_{i=0} N_i = 1$. Furthermore, let $\tau \leq 1$ denote the fraction of the high type firms searching in submarket $1_2$ (for employed workers), and $1 - \tau$ the fraction searching in market $0_2$ (for unemployed workers). Similarly, let $\kappa$ denote the fraction of unemployed workers searching for high type firms, and $1 - \kappa$ the fraction searching for low type firms. The flow equation for $N_0$ is defined as

$$N_0[(1 - \kappa)p_{01} + \kappa p_{02}]$$

$$= (s + \delta)(N_1 + N_2).$$

The flow equations for $N_1$ and $N_2$ are defined analogously. Let $k$ denote the number of firms in the economy. Labor market tightness in submarket $0_1$ is then given by

$$\theta_{01} = (1 - \alpha) \frac{v_{01}}{(1 - \kappa)N_0}. \quad (8)$$

Labor market tightness in submarkets $0_2$ and $1_2$ are defined analogously.

**DEFINITION 1:** The equilibrium is a vector of asset values $M_0$, $M_1$, and $M_2$, two fractions $\tau$ and $\kappa$, and a number $k$ such that the following requirements are satisfied.

---

3 In addition, the wage must be below (above) $W'$ if the firm wants to attract unemployed (employed) workers. As this constraint never binds, it is ignored; see Garibaldi and Moen (2009).
(i) **Optimal search:** the asset values \( M_0, M_1, \) and \( M_2 \) are given by equations (3), (5), and (6).

(ii) **Optimal allocation on submarkets:** either \( \pi_{12} = \pi_{02} = \pi_2 \) or \( \kappa = \tau = 0 \).

(iii) **Zero profit ex ante:** \( (1 - \alpha)\Pi_1 + \alpha\Pi_2 = K \).

(iv) **Aggregate consistency:** The flow conditions and the definition of \( \theta_i \) are satisfied.

D. Properties of Equilibrium

An important consideration is whether the 02 market will open up (stairway to heaven), in which case we refer to a mixed job ladder. If the 02 market does not open up, we refer to a pure job ladder. As the next proposition shows, whether we have a mixed or pure job ladder depends on parameter values. However, the wage structure in the different submarket is always the same:

**PROPOSITION 2:** a) For low values of \( \alpha \), all the three submarkets are active, and we have a mixed job ladder equilibrium. For high values of \( \alpha \), only the 01 and the 12 markets are active, and we have a pure job ladder equilibrium.

b) The following is always true: \( W_{01} < W_{02} < W_{12} \) and \( p_{01} > p_{02} > p_{12} \).

Suppose \( \alpha \) is low, so that there are few high type firms and many workers employed in low type firms. By offering a wage slightly above \( y_1 \), high type firms fill their vacancies quickly, grow quickly, and obtain a large profit. Hence they have no incentives to search for unemployed workers, and the economy is in a pure job ladder equilibrium. As \( \alpha \) grows, the 12 market becomes more crowded with high type vacancies relative to workers searching on the job, and profits fall. At some point the 02 submarket opens up, and the economy is in a mixed job ladder equilibrium. In this kind of equilibrium, a fraction \( \tau \) of the high type firms search for unemployed workers, and a fraction \( \kappa \) of the unemployed workers search for high type firms. The fractions \( \tau \) and \( \kappa \) are determined so that high type firms are indifferent between searching for employed and unemployed workers, while unemployed workers are indifferent between searching for high type and low type firms.

The concavity of the matching function implies that a high matching rate for agents on one side of the market (say firms) implies a low matching rate for agents on the other side of the market. Thus, it is efficient to let agents with a relatively low opportunity cost of waiting (employed workers and low type firms) search for agents with a high opportunity cost of waiting (unemployed workers and high type firms), and let the former match quickly and the latter slowly. Given the constraints imposed by the stocks of workers and firms, this is also how resources are allocated in equilibrium. In equilibrium, this is obtained by paying employed (patient) workers a high wage when matched (\( W_{12} \) is the highest wage in the economy), while the low type (patient) firms pay a relatively low wage for workers (\( W_{01} \) is the lowest wage in the economy). The wage in the 02 submarket is intermediate.

II. The Increase in Productivity in Aggregate Labor Markets

The features of the pure and mixed job ladder equilibria can best be understood with the help of numerical simulations, obtained by a simple search routine described in Garibaldi and Moen (2009). In the specification of the model presented in this section, we assume that the convexity of the vacancy cost is extreme so that each firm can post at most a maximum number of vacancies \( v \).

The main objective of the simulations is to show the mechanics of the model for different values of \( \alpha \). As \( \bar{y} = (1 - \alpha)y_1 + \alpha y_2 \), an increase in \( \alpha \) is akin to an increase in average productivity. The basic charts of the simulations are provided in figures 1 and 2. First note that when \( \alpha = 0 \) or 1, the model collapses to the traditional matching model without on the job search (Pissarides 2000). As expected, the transition rate from unemployment to employment is higher and unemployment lower when \( \alpha = 1 \) than when \( \alpha = 0 \). (In Figure 1 unemployment falls from 0.0968 to 0.083 as \( \alpha \) increases from 0 to 1.) We refer to this as a pure productivity

---

4 The rest of the parameters are as follows. The interest rate \( r \) is 0.01, the separation rate \( s \) is 0.04, while the firm exit rate is 0.02. The baseline productivity \( y_1 \) is normalized to one while the high type firm productivity is 1.08. The outside income is [0.55] and the marginal cost of vacancies is [0.2]. The matching function is Cobb-Douglas with sharing parameter equal to 0.5 and constant parameter \( A = 1 \).
relatively low levels of workers, tending to increase unemployment. For higher values of \( \alpha \), the productivity effect dominates, as the share of high productivity firms increases toward one, the pure productivity effects dominate, and unemployment falls.

Finally, the nonmonotonic behavior of entry deserves a few comments. When \( \alpha \) is low, the value of a high type firm (given by \( 7 \)) is extremely high since this type of firm grows so quickly. Hence, the number of firms increases rapidly in \( \alpha \). However, for higher value of \( \alpha \), the marginal value of a high type firm is lower, and fewer firms enter. This explains the hump-shaped form of the number of firms in the economy.

REFERENCES


AUTHOR, PLEASE ANSWER ALL QUERIES (numbered with “AQ” in the margin of the page).

<table>
<thead>
<tr>
<th>AQ#</th>
<th>Question</th>
<th>Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>“For wages below W’,” incomplete sentence.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>It might be clearer if the titles for the figures were “Effect of Average Productivity on Stocks and Job Finding Rates” (Figure 1) and “Effect of Average Productivity on Valve Functions, Flows and Entry” (Figure 2).</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>Missing value for y axis in Figure 2.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>“Given by 7”? Do you mean given by equation (7)?</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>Your paper slightly exceeds the length limit for your session, which is five pages. Please consider shortening the paper or moving some materials to an online Appendix.</td>
<td></td>
</tr>
</tbody>
</table>
COPYRIGHT TRANSFER AGREEMENT

From: The American Economic Association

Fax: +1 412-431-3014

The American Economic Review
2403 Sidney Street, Suite 260
Pittsburgh, PA 15203

To: Author (please print name here)

The American Economic Association (hereinafter Association) is pleased to have the opportunity to publish your manuscript in the American Economic Review. In order that the Association, as Publisher, may obtain copyright protection for the contents of the Journal, it is necessary for you to execute this formal transfer of your copyrights in this manuscript to the Association.

The Association acknowledges the receipt of your manuscript titled ___________________, to appear in the May 2010 issue of AER.

Consent to Publish

In consideration of the publication by the Association of the above-named manuscript, the undersigned as Author(s) transfer(s) exclusively to the Association all rights, title and interest defined by the Copyright Law of the United States in and to the above-named manuscript in its entirety, including all subsidiary rights. The rights transferred herein shall remain the property of the Association for the full duration of these rights under the Copyright Law of the United States. If it should become necessary, the Author(s) agree(s) to assist the Association in registering and enforcing the Copyright in the name of the Association. The Association shall have the right to publish the above-named manuscript in print, sound or video recordings, magnetic media (i.e., computer disk, CD-ROM, etc.), electronic media (including transmission via the Internet, or any other computerized communication network), or any other technology for publication of this work which may hereinafter be developed.

The Association, in turn, grants to the Author(s) the right to republication in any work in any form, including digital repositories in universities and other institutions subject only to giving proper credit of copyright. The Association further grants to the Author(s) the right to distribute the above-noted work in any classroom in which he or she is a teacher, subject only to the Author(s) giving proper credit in any such derivative work and on any copies distributed for classroom use. Proper notice may be given as follows: [Copyright ______, American Economic Association; reproduced with permission of the American Economic Review].

Permission to Reprint Policy

The Author(s) may specify the degree of access to which the Association grants others the right to reproduce the Author(s)' material. Check one:

_______ Implicit consent: Grants anyone permission to reprint in all places in all forms provided that the appropriate copyright information is included and the Association is notified that the work is being reprinted.

_______ Explicit consent: Requires direct consent of the Author(s) and the Association before any republication is allowed. The republisher must obtain from the Author(s) permission to reprint all or any major portion of the Author(s)' manuscript. Author(s) may charge a fee for reprint or translation rights.

Rights to translate are retained by the Author(s) and dealt with on a case by case basis.

Warranty of Authorship

The Author(s) warrant(s) that the above-named manuscript is his or her own original work of authorship and has not been published previously. If any material included by the Author(s) in the above-noted manuscript (including tables, charts, or figures) is the work of another author or is otherwise under prior copyright protection by another proprietor, the Author(s) undertake(s) to obtain permission from that copyright proprietor for the inclusion of such material in this manuscript to be published by the Association. The Author(s) further agree(s) to save and hold the Association harmless in any suit for infringement arising from the Author(s)’ unauthorized use of copyrighted material. The Author(s) agree(s) to submit to the Editor of the Journal of the Association to whom the manuscript has been submitted, copies of all letters of permission to include copyrighted material of another author included in the subject manuscript by this Author or material written by Author(s) that is under prior copyright protection by another proprietor.

The Author(s) further warrant(s) that this manuscript was not written as part of his or her official duties as an employee(s) of the United States government. Since copyright protection is not available for a work of the United States government, the Author(s) agree(s) to disclose fully to the Association the circumstances of federal employment which might invoke this bar to copyright protection of the manuscript by signing below to confirm the author warranties..

The Author(s) further warrant(s) that this manuscript was not written as an employee so as to constitute a work-for-hire in which the ownership of the copyright is in that employer.

Please sign and date this agreement. Return one copy to the Editor of the American Economic Review promptly and retain one copy. A manuscript for which there is no valid Copyright Transfer Agreement cannot be published.

Accepted and approved: ________________________________

Author(s)

Government Employees please sign here: ________________________________ Date: ________________________________

For the American Economic Association and the American Economic Review:
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>MEANING</th>
<th>EXAMPLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>/ \ /</td>
<td>delete</td>
<td>take \ out</td>
</tr>
<tr>
<td>close up</td>
<td>close up</td>
<td>per cent</td>
</tr>
<tr>
<td>delete and close up</td>
<td>delete and close up</td>
<td>remove</td>
</tr>
<tr>
<td>insert something here</td>
<td>insert something here</td>
<td>something is missing</td>
</tr>
<tr>
<td>space</td>
<td>space</td>
<td>too close</td>
</tr>
<tr>
<td>space evenly</td>
<td>space evenly</td>
<td>these consistently</td>
</tr>
<tr>
<td>let stand</td>
<td>let stand</td>
<td>ignore marks and leave as was</td>
</tr>
<tr>
<td>transpose</td>
<td>transpose</td>
<td>this[backwards]is</td>
</tr>
<tr>
<td>used to separate 2 or more marks in margin</td>
<td>used to separate 2 or more marks in margin</td>
<td>^ / ^</td>
</tr>
<tr>
<td>center</td>
<td>center</td>
<td>[this should center[</td>
</tr>
<tr>
<td>set farther to the left</td>
<td>set farther to the left</td>
<td>[move left</td>
</tr>
<tr>
<td>set farther to the right</td>
<td>set farther to the right</td>
<td>move right]</td>
</tr>
<tr>
<td>align horizontally</td>
<td>align horizontally</td>
<td>align with surrounding text</td>
</tr>
<tr>
<td>align vertically</td>
<td>align vertically</td>
<td>align with surrounding text</td>
</tr>
<tr>
<td>move to next line</td>
<td>move to next line</td>
<td></td>
</tr>
<tr>
<td>begin new paragraph</td>
<td>begin new paragraph</td>
<td></td>
</tr>
<tr>
<td>spell out</td>
<td>spell out</td>
<td>set PA as Pennsylvania</td>
</tr>
<tr>
<td>set in capitals</td>
<td>set in capitals</td>
<td>ALL CAPS</td>
</tr>
<tr>
<td>set in small capitals</td>
<td>set in small capitals</td>
<td>SMALL CAPS</td>
</tr>
<tr>
<td>set in lowercase</td>
<td>set in lowercase</td>
<td>lower case</td>
</tr>
<tr>
<td>set in italic (underline the text)</td>
<td>set in italic (underline the text)</td>
<td>italic</td>
</tr>
<tr>
<td>set in roman</td>
<td>set in roman</td>
<td>roman or regular</td>
</tr>
<tr>
<td>set in bold (squiggly underline of text)</td>
<td>set in bold (squiggly underline of text)</td>
<td>BOLD</td>
</tr>
<tr>
<td>hyphen-used to join words and to separate syllables</td>
<td>hyphen-used to join words and to separate syllables</td>
<td></td>
</tr>
<tr>
<td>en dash—a connection between two things</td>
<td>en dash—a connection between two things</td>
<td>2006–2007</td>
</tr>
<tr>
<td>em dash—a connection between two things</td>
<td>em dash—a connection between two things</td>
<td></td>
</tr>
<tr>
<td>superscript or superior</td>
<td>superscript or superior</td>
<td>E=MC^2</td>
</tr>
<tr>
<td>subscript or inferior</td>
<td>subscript or inferior</td>
<td>( H_2O )</td>
</tr>
<tr>
<td>centered</td>
<td>centered</td>
<td>for a centered dot in ( p \hat{\lambda} q )</td>
</tr>
<tr>
<td>comma</td>
<td>comma</td>
<td>red, white, and blue</td>
</tr>
<tr>
<td>apostrophe</td>
<td>apostrophe</td>
<td>my sister’s friend’s investments</td>
</tr>
<tr>
<td>period</td>
<td>period</td>
<td>the end.</td>
</tr>
<tr>
<td>semicolon</td>
<td>semicolon</td>
<td>he said; she said</td>
</tr>
<tr>
<td>colon</td>
<td>colon</td>
<td>what follows proves: clarifies</td>
</tr>
<tr>
<td>quotations marks</td>
<td>quotations marks</td>
<td>“the economist”</td>
</tr>
<tr>
<td>parentheses</td>
<td>parentheses</td>
<td>(like this)</td>
</tr>
<tr>
<td>brackets</td>
<td>brackets</td>
<td>[like this]</td>
</tr>
<tr>
<td>wrong font</td>
<td>wrong font</td>
<td>wrong size or style</td>
</tr>
</tbody>
</table>