On the Dispersion of Skill and Size in Active Management: Multi-Agent Dynamic Equilibrium with Endogenous Information

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Abstract

What is the optimal size of the active portfolio management industry? How does the distribution of skill and wealth across investors within industry affect asset prices and information aggregation? This paper studies these and closely related questions in a dynamic rational expectations equilibrium (REE) economy with endogenous information. The dispersion of skill and capital arise endogenously in our infinite horizon model with ex-ante identical investors with constant relative risk aversion (CRRA). Besides new theoretical implications, our model links private information (unobservable) to capital allocation (observable), and thereby brings the predominantly theoretical REE literature one step closer to the econometrician.

Keywords: information choice and aggregation, active portfolio management, dynamic equilibrium, wealth dispersion, investor skill

JEL Classification: D53, D82, G11

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1 Introduction

The investment management industry has faced tremendous developments over the past few of decades. Since World War II, institutional ownership has increased sharply.[1] However, the fraction of institutionally held equity that is actively managed has decreased during this time. Stambough [2014] documents that this share decreased from about 100% in 1980 to around 83% in 2012. In order to understand what awaits this industry in the future, we present a micro-founded model in which the scope and capital allocation of the active management industry arise endogenously. Aside from its theoretical contribution, this paper provides empirical implications that contribute to the testability of related rational expectations equilibrium (REE) models.

We model the active portfolio management industry by extending a Verrecchia [1982] model with (i) wealth effects generated by CRRA preferences and (ii) multiple trading rounds. In contrast to papers such as Berk and Green [2004], we interpret "skill" as the investor's overall ability to enhance portfolio returns that can be improved by investing in costly information. Investors of our model are ex-ante identical,[2] but initial luck can create systematic performance differences. As a consequence, the market would perceive some investors to be more skilled then others, despite their initial similarity.

The equilibrium distribution of wealth is pinned down by two opposing forces. First, due to CRRA preferences, information yields increasing returns to scale:[3] Richer agents take on more risk by having larger dollar amounts invested in risky securities. Information regarding the performance of these risky investments is thus more useful to the affluent agent. Naturally, a feedback loop arises in which wealthier agents buy more information, obtain higher (expected) returns on their portfolio, and get comparatively even richer. This capital inequality enlarging force ensures that even small initial wealth differences can snowball into large long-term heterogeneities in capital and (perceived) skill among investors.

Whilst a continuous growing dispersion in wealth is theoretically convincing, some data-driven concerns persist: First, it is surprising that despite the returns to scale argument in the previous paragraph, there is no natural monopolist in the active portfolio management industry. Second, empirical studies such as **?** find decreasing returns to scale on both the fund and industry level, instead of the increasing returns to scale generated by the mechanism previously presented.

[1] French [2008] estimates that after World War II, 90% of U.S. equity was directly held by individual investors and that this fraction has fallen to around 20% recently.

[2] Except for noise traders, who because of endowment shocks only.

[3] See also Peress [2003] and Nieuwerburgh and Veldkamp [2010]

Without making additional assumptions in our model, however, a countervailing force prevents capital inequality across investors to grow indefinitely. When investors trade large amounts, they leave an informational "footprint" that allow outsiders to free-ride on insiders' private information. This is a common feature in the REE literature following Grossman and Stiglitz [1980], but is especially applicable to the richer agents in our setup. The larger the capital inequality, the stronger this free-riding effect prevents wealth dispersion from growing even further. These decreasing returns to scale could dominate the increasing returns to scale in the previous paragraph. To our knowledge, this is the first paper to endogenise the distribution of wealth and skill by combining the informational returns to scale and free-riding effect.

We split the further analysis of our economy into two parts. In the first part, we take the distribution of wealth as exogenous. Wealthier agents spend disproportionally more on information. For a fixed level of aggregate wealth, total expenditures on information are therefore increasing in capital inequality. The expected growth of actively held wealth is higher and prices are more informative when wealth dispersion is larger. Consequently, trading in risky assets is more attractive and volatility of returns and expected returns on stocks are lower.

In the second part, we study an infinite horizon equilibrium and endogenise the distribution of wealth—the only state variable in our model.[4] The larger the noise in the economy,[5] the weaker is the informational free-riding effect that is responsible for decreasing returns to scale. Investors then leave a smaller informational footprint which disproportionally benefits the rich and results in a larger *dispersion* in wealth. A similar mechanism is in place when the cost for information decrease (for example when cheaper technologies to collect data are available). While the free-riding effect remains the same, the informational returns to scale effect is larger, leader to a larger equilibrium in capital inequality and dispersion of investor skill.

Endogenizing the distribution of wealth is not only a theoretical contribution, but also an aspect that could be exploited by the econometrician to test REE models. Specifically, our model links private information allocation to wealth dispersion. The latter variable is typically easier to observe for the econometrician, who could therefore use wealth dispersion as an instrumental variable to measure private information.

This method is best illustrated with an example. Consider again Stambough [2014], who explains the decrease in size for active portfolio management by a decrease in noise traders (measured by

[4] Note that the distribution of skill is directly to wealth

[5] Noise is measured as the volatility of the endowment shock of the noise traders.

individual stock ownership). Assume the scope for active management is represented by the growth of wealth of informed speculators. According to our model, a reduction in noise trading not only decreases the scope for active management, but also the dispersion in wealth and skill, which are typically observable. Hence, we could test Stambough [2014]'s assertion by verifying whether or not wealth and skill dispersion decreases when institutional ownership decreases. For related papers, the linkage between information and wealth dispersion could potentially provide similar additional testable conditions which would improve the testability of REE models.

This paper also questions the empirical testability of person-specific portfolio management "talent". In contrast to papers such as Berk and van Binsbergen [2013], we claim that capital drives labor productivity. Hence, even if managers have identical ex-ante talent, they can perform differently in equilibrium. Using the dollar "value added" as suggested by this literature would measure the joint labor and capital productivity, the latter stemming from increasing risk absolute risk tolerance and the willingness to acquire information. Putting things simply, if manager A has a value added of $1M and manager B a value added of $50K, then it could even be that manager B is more talented, if he by a few lucky initial draws managed to raise more capital and invests in more performance-enhancing resources.

This paper contributes to multiple streams of the literature. First, although we do not model institutional frictions explicitly, this paper provides new insight on the extensive literature on active portfolio management, beyond the scope of this paper. We relate to Pastor et al. [2014] and Stambough [2014] by endogenising the scope for active management in a micro-founded REE model with wealth effects and add a new testable implication. This paper generates decreasing returns to scale in the asset management as in Stambough [2014], Berk and van Binsbergen [2013] and Berk and Green [2004] without explicitly modeling to endogenise fund flows. By doing so, we eliminate the "need" to model institutional frictions to explain this empirical observed phenomenon.

Next, this paper contributes to the literature on capital inequality. In contrast to Kacperczyk et al. [2014], we endogenise not only capital inequality, but also investor skill. While we do not make welfare statements, we contribute to the discussion on capital (income) inequality that gained momentum following Piketty [2003] and Piketty and Saez [2003]. Specifically, we show a benefit of capital inequality: In our model, dispersion in wealth inequality increases the informativeness of equilibrium prices, which due to the free-riding effect is especially beneficial to poorer investors.

Last, we contribute to the growing stream of papers that examine rational expectations equilibria outside the CARA/normal framework developed by Grossman and Stiglitz [1980], Hellwig [1980] and others. Where works such as Barlevy and Veronesi [2000] and Breon-Drish [2015] focus on different distributional assumptions, we follow the path of Peress [2003] and Bernardo and Judd [2000] by changing the preference structure. We extend the model of Breugem and Buss [2018], which can handle any preference and distributional assumption, to a dynamic framework with heterogeneous investors.[6] By doing so, we aim to integrate the highly stylised REE literature (where it is hard to model frictions and different preferences) with the traditional asset pricing literature (where there is learning from prices).

The remainder of the paper is organized as follows: Section 2 describes the details of the model. Section 3 discusses the (numerical) solution technique. We break the analysis of our results into two steps. Sections 4-6 study the static implication of the model for a fixed distribution of wealth. Sections 7-8 endogenise the distribution of capital and study implications for asset pricing. Section 9 explains empirical implications and Section 10 discusses some limitations and extensions of our model. Robustness checks and proofs will be available as an internet appendix.[7]

2 The Economy

In this chapter we present the assumptions of our model. In brief, the setup resembles a dynamic Verrecchia [1982] model with CRRA preferences. Our setup and discretization procedure follows Breugem and Buss [2018].

2.1 Agents and assets

Our setup consists of a multi-period production economy with time $t \in \{0, .., T\}$ and with discount rate $\beta < 1$. We are mainly interested in the cases $T = 2$ and $T \to \infty$. The economy is populated with two *ex-ante identical* competitive rational agents. Regarding notation, we present all variables without agent-specific subscripts but instead marked with an asterix to indicate "the other agent" when explicit differentiation is needed. Agents are endowed with identical initial wealth W_0 and

[6] Our method could potentially even further improved by making use of the efficient incomplete market algorithm of Dumas [2012]

[7] The internet appendix will be posted online when available, and is can be requested from the author at any time in preliminary form.

maximize their power utility with relative risk aversion γ over their final period consumption c_T. Aggregate wealth in the economy is denoted by $\overline{W}_t = W_t + W_t^*$ and the wealth share of an agent by $\omega_t = W_t/\overline{W}_t$.

In each trading round, agents can invest their wealth across two short-lived assets. The first asset ("the bond") is risk-free and yields a unit payoff with certainty. The second asset ("the stock") is risky, has inelastic supply Z and produces a risky payoff $D_t \in \{D_H, D_L\}$ with $\mathbb{P}[D_H] = \mathbb{P}[D_L] = \frac{1}{2}$.[8,9] We denote the fraction of an agent's wealth invested in riskless and risky asset by ϕ_t and λ_t respectively.

2.2 Purchase of private information

During each trading round, agents can purchase a costly signal that is (privately) informative regarding the payoff realization of the risky asset. This information is useful to agents, as it allows them to enhance trading profits by investing more (less) in the stock upon receiving positive (negative) news. Information expenditures are deducted from an agents investable wealth.[10]

Larger expenditures on information result in more accurate private information: Specifically, upon investing in information, agents receive a private signal $y_{i,t} \in \{y_H, y_L\}$. The "quality" of this signal ρ_t describes the probability that the signal is identical risky asset's payoff realization in the next period:

$$\rho_t = \mathbb{P}[D_H|y_H] = \mathbb{P}[D_L|y_L] \in \left[\frac{1}{2}, 1\right)$$

Limit cases are (i) $\rho_t = \frac{1}{2}$ in which the signal is not informative and (ii) $\rho_t \to 1$ which corresponds to perfect foresight. Signals of higher quality are better predictors of future returns, but are more costly to acquire. I assume that the cost of information κ_t is an increasing and convex function of its precision:

$$K_t \quad (\rho_t) = \quad \Xi \times \Gamma(\rho_t)^\alpha$$

Where $\Gamma(\rho_t) = \frac{1}{\rho_t(1-\rho_t)} - 4$ represents the precision of the signal in excess of the precision of an uninformative signal.[11] Note that $K\left(\frac{1}{2}\right) = 0$ and $K(1) = \infty$ which ensures that uninformative

[8]We denote the probability of the realization of a random variable simply by $\mathbb{P}[x]$ instead of $\mathbb{P}[X = x]$ where X is a random variable and x a realization.

[9]To mimic a binomial tree structure with short-term securities, we assume that Z is proportional to the aggregate wealth in the economy.

[10]see e.g. Verrechia (1982)

[11]Recall that the precision of a bernouilli random variable with probability ρ_t is given by $\frac{1}{\rho_t(1-\rho_t)}$. The precision of an uninformative signal (setting $\rho_t = \frac{1}{2}$) is 4.

signals are free and perfectly information is infinitely costly. Setting the scaling parameter α above unity ensures this cost function is increasing and convex.

In order to make a meaningful comparison across time, we need to ensure that the information cost function has a constant influence on our result across different periods. We therefore let the scaling parameter $\Xi = \xi \, \overline{W}_t$ be proportional to the total wealth in the economy, so that only wealth inequality matters for information allocation. I denote by $\kappa_t = \xi \times \Gamma\left(\rho_t\right)^{\alpha}$ the corresponding normalized cost of information.

2.3 Learning from prices

Agents with information desire to act on it by selling stock upon a negative signal and buying stocks upon a positive signal. Since the total supply of assets is fixed at Z, positive (negative) news will therefore have a increasing (decreasing) effect on the equilibrium price of the stock. Any rational agent can use this information to learn about the private signal realization of other informed speculators. A posterior belief π_t consists hence aggregates private information and public information contained in equilibrium prices.

In the setup we introduced so far, equilibrium prices would be fully revealing: Each agent could simply calculate the other agent's demand by solving that agent's optimization problem and compute corresponding equilibrium prices for each possible signal realization. The observation of an equilibrium price now uniquely pins down the realization of other agent's private signal.[12] Since no private information can be kept secret in this situation, speculative trade does not occur (see Milgrom and Stokey [1982]), and the value of private information equals zero. Grossman and Stiglitz [1980] show that this in this case no competitive equilibrium exists.[13]

In line with the common practice in the literature, we introduce noise traders (denoted with suprascript "$^{\circ}$") to prevent equilibrium stock prices from fully aggregating information. In our model, noise traders have identical (CRRA) preferences to rational agents, but (i) are myopic, (ii) do not learn from prices, (iii) do not acquire private information and, (iv) are endowed with a random amount of wealth. This random wealth is unobservable to our informed agents and is proportional to the aggregate wealth

[12] The possibility to *invert* the price function is not restricted to our distributional assumptions

[13] Specifically, if no agent acquires private information, prices are uninformative so that each agent unilaterally wishes to deviate (by buying information). But if both agents do this, prices are again fully revealing and the loop continues. Hence no equilibrium exists

in the economy: $W_t^{\circ} = \varpi_t \overline{W}_t$ with $\varpi_t \sim \mathcal{N}(\mu_{\varpi}, \sigma_{\varpi})$. Since the absolute risk tolerance of the noise trader is proportional to his wealth, his demand for stocks is increasing in ϖ_t.[14]

The presence of noise traders somewhat hinders the ability of rational agents to learn from equilibrium prices. For example, the observation of a high equilibrium price is likely to be consistent with a positive signal other rational agent, even though it could also be a result of a positive wealth shock of the noise traders. Of course, the observation of a high equilibrium price is more likely to be consist with a positive signal of the other agent than with a negative one, and therefore agents still learn in the presence of noise. The next section explains this *learning from prices* in detail.

2.4 Timing

Within each period, each agent faces the joint problem of information acquisition and portfolio choice. The timing within each period $t < T$ consists of four sub periods:

1. The risky asset's payoff D_t and noise trader wealth shock ϖ_t are realized; each investor's wealth W_t is computed.
2. Agents choose the quality ρ_t of their respective private signal y_t. The signal's cost $K_t(\rho_t)$ is deducted from the agent's wealth W_t.
3. The private signal y_t is revealed and agents allocate their remaining wealth across risky and riskless assets. To learn from equilibrium prices S_t in addition to from y_t alone, agents submit a demand schedule as a function of equilibrium price S_t.
4. A Walrasian auctioneer determines S_t by matching supply and demand, which pins down portfolio holdings of all agents.

In the last period, when $t = T$, agents consume their final wealth $c_T = W_T$.

3 The Equilibrium

This section introduces our solution method in detail. In essence, as in Breugem and Buss [2018], we solve a large fixed-point problem numerically over a grid of state variables at each point in time.

[14]In case $\varpi_t < 0$, the demand shock of the noise trader is negative.

3.1 Single-agent problem

Denote by $R_{t+1} = \frac{D_{t+1} - S_t}{S_t}$ the return on the risky security. Following the assumptions made previously, the value function of each rational agent during trading round $t < T$ is denoted by:

$$V_t \left[W_t\right] = \max_{\rho_t, \{\lambda_t\}, \{C_t\}} \mathbb{E} \left[U \left[C_t\right] + \beta \, V_{t+1} \left[W_{t+1}\right]\right] \tag{1}$$

Note that consumption C_t depends on the signal realization and therefore cannot be taken outside of the expectation operator. As a terminal condition, we impose that agents consume their final-period wealth:

$$V_T \left[W_T\right] = U \left[C_T\right] = \frac{W_T^{1-\gamma}}{1-\gamma}$$

The objective problem should be solved subject to the motion of wealth condition:

$$W_{t+1} = W_t \, \left(1 + \lambda_t \, R_{t+1}\right) - K_t - C_t = W_t \, \left(1 + \lambda_t \, R_{t+1} - \frac{\kappa_t}{\omega_t} - c_t\right) \tag{2}$$

Where c_t is the fraction of an agent's wealth that is consumed. Now, after substituting the budget constraint and the expectation operator in terms of probabilities, the Lagrangian can be written as:

$$\mathcal{L}_t = \sum_{\zeta_t} \sum_{D_{t+1}} \Pi_t \, \left(U \left[C_t\right] + \beta \, V_{t+1} \left[W_{t+1}\right]\right)$$

Where $\Pi_t = \mathbb{P} \left[\zeta_t, D_{t+1}\right] = \pi_t \, \mathbb{P} \left[\zeta_t\right]$ represents the ex-ante probability of each state of nature, where $\zeta_t = \{y_t, y_t^*, \varpi_t\}$. Note that variables λ_t and S_t have different realizations for each of ζ_t. I next substitute (2) in the budget equation and use the homogeneity property of the value function to write:[15]

$$\mathcal{L}_t = \sum_{\zeta_t} \sum_{D_{t+1}} \Pi_t \, W_t^{1-\gamma} \, \left(U \left[c_t\right] + \beta \, V_{t+1} \left[1 + \lambda_t \, R_{t+1} - \frac{\kappa_t}{\omega_t} - c_t\right]\right)$$

Due to the nature of our setup, total wealth only appears as a multiplicative constant in the Lagrangian. First order conditions with respect to risky asset holdings should computed for all possible values of ζ_t and are:[16]

[15]The proof follows by the induction and is available on request. In short, we show that $V_t \left[W_t \, X\right] = W_t^{1-\gamma} \, V_t \left[X\right]$ for all t

[16]For the myopic noise trader, (4) simplifies to:

$$0 = \sum_{D_{t+1}} \pi_t \, R_{t+1} \, \left(1 + \lambda_t^{\circ} \, R_{t+1}\right)^{1-\gamma} \tag{3}$$

$$0 = \sum_{D_{t+1}} \pi_t \, R_{t+1} \, V'_{t+1} \left[1 + \lambda^*_t \, R_{t+1} - \frac{\kappa_t}{\omega_t} - c_t \right] \qquad \forall \zeta_t \tag{4}$$

Holding consumption and information spendings constant, a higher investment in risky securities decreases the available wealth invested in the riskless storage technology. The values λ^*_t that pin down (4) determine the optimal risk profile of the investor. Given this optimal risk profile, the investor needs to decide how much to consume and how much to save. The optimal fraction of consumed wealth c^*_t follows from:

$$0 = \sum_{D_{t+1}} \pi_t \, W_t^{1-\gamma} \left(U' \left[c^*_t \right] - \beta \, V'_{t+1} \left[1 + \lambda_t \, R_{t+1} - \frac{\kappa_t}{\omega_t} - c^*_t \right] \right) \qquad \forall \zeta_t \tag{5}$$

The higher the discount factor β, the higher future periods are important to the investor, and the lower the share of wealth consumed. Finally, the first order condition with respect to information quality can be computed by taking the following (total) derivative:

$$0 = \sum_{\zeta_t} \sum_{D_{t+1}} \left(\frac{\partial \Pi_t}{\partial \rho_t} \left(U \left[c_t \right] + \beta \, V_{t+1} \left[W_{t+1} \right] \right) + \Pi_t \left(\frac{\partial c_t}{\partial \rho_t} U \left[c_t \right] + \beta \left(\frac{\partial \lambda_t}{\partial \rho_t} R_{t+1} - \frac{1}{\omega_t} \frac{\partial \kappa_t}{\partial \rho_t} \right) V'_{t+1} \left[W_{t+1} \right] \right) \right) \tag{6}$$

To shorten notation, we denote $W_{t+1} = 1 + \lambda_t \, R_{t+1} - \frac{\kappa_t}{\omega_t} - c_t$. The first part of the right hand side of the equation denotes the changes in utility due to changes in information, keeping action fixed. The second term denotes the contribution in utility from a change in actions (i.e., investments and information choice) keeping probabilities fixed. Another way to write the above equation is to decompose the equation into the marginal benefit MB_ρ and marginal cost MC_ρ of (private) information quality:

$$MB_\rho \;\; = \;\; \sum_{\zeta_t} \sum_{D_{t+1}} \left(\frac{\partial \Pi_t}{\partial \rho_t} \left(U \left[c_t \right] + \beta \, V_{t+1} \left[W_{t+1} \right] \right) + \Pi_t \left(\frac{\partial c_t}{\partial \rho_t} U \left[c_t \right] + \beta \, \frac{\partial \lambda_t}{\partial \rho_t} R_{t+1} V'_{t+1} \left[W_{t+1} \right] \right) \right)$$

$$MC_\rho = \sum_{\zeta_t} \sum_{D_{t+1}} \beta \, \Pi_t \left(\frac{1}{\omega_t} \frac{\partial \kappa_t}{\partial \rho_t} \right) V'_{t+1} \left[W_{t+1} \right] \tag{7}$$

The optimal level of information quality ρ_t should be chosen such that $MB_\rho = MC_\rho$.[17] Figure 1 depicts the two curves. At $\rho_t = \frac{1}{2}$, the derivative with respect to ρ_t of MB_ρ is strictly positive while those of MC_ρ is zero. The marginal benefit of information is increasing due to the increasing returns to scale of information (see e.g. Nieuwerburgh and Veldkamp [2010]).[18]

Finally, given optimality of $\rho_t, \{\lambda_t\}$, and $\{c_t\}$, agents should invest their remaining wealth into riskless bonds:

[17] normalized by $\left(\overline{W}_t\right)^{1-\gamma}$

[18] A sufficiently convex cost function hence guarantees a positive optimal level of information. In our dynamic economy, the existence of a unique solution is crucial. therefore, we pick α large enough so that the cost function is sufficiently convex to guarantee a unique optimal level of ρ_t.

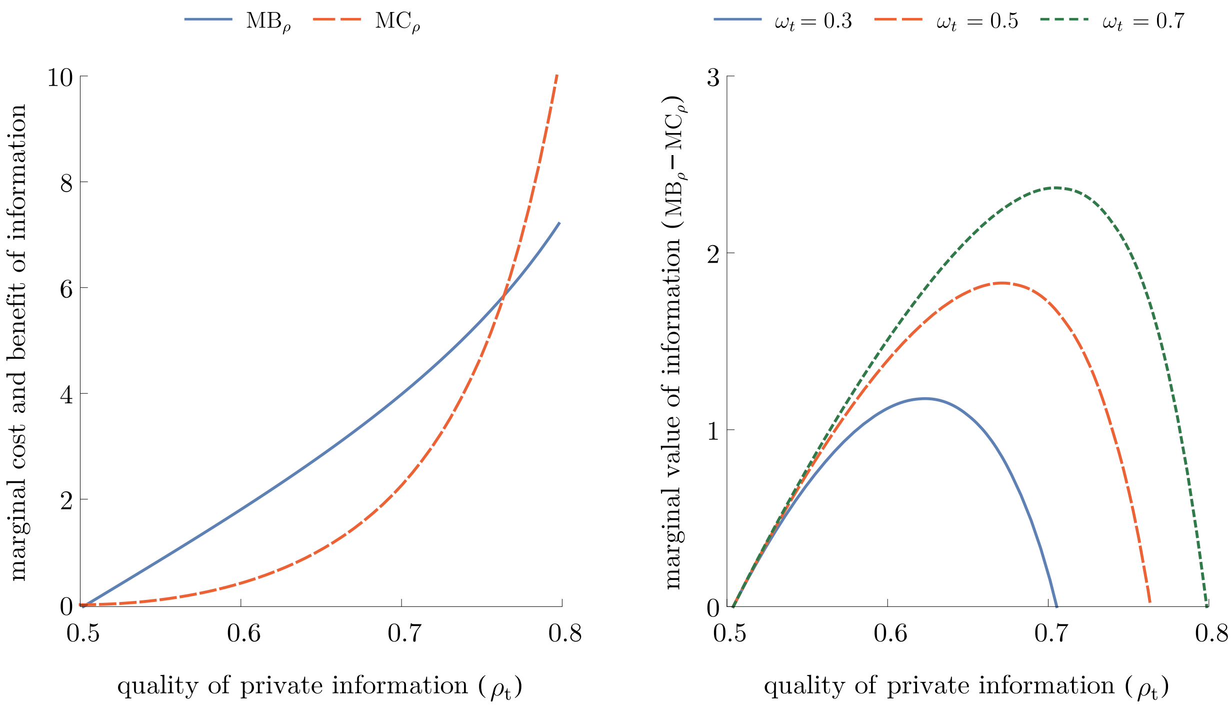

Figure 1: Optimal purchase of private information. LEFT FIGURE (A): Marginal benefit and marginal cost of private information (for $\omega_t = \frac{1}{2}$). A sufficiently convex information cost function guarantees an optimal quality of private information ρ_t. Optimality arises when the MB_ρ and MC_ρ curve intersect, which occurs at $\rho_t = 0.73$ in the presented scenario. Note the intersection of the MB_ρ and MC_ρ curves at $\rho_t = 0.5$ arises due to a local minimum. The area between the MB_ρ and MC_ρ curves (for $0.5 \le \rho_t \le 0.73$) represent the agent's surplus due to private information acquisition. RIGHT FIGURE (B): The marginal value of private information is the difference between the MB_ρ and MC_ρ curves. The surface below the graphs represents the agent's surplus due to private information acquisition. The optimal level of information and the investor's surplus is increasing an agent's (relative) wealth, generated by the increasing returns to information under CRRA utility.

$$\phi_t = 1 - \lambda_t - \frac{\kappa_t}{\omega_t} - c_t \qquad \forall \zeta_t \tag{8}$$

3.2 Market Clearing

Aggregate demand for stocks must match with aggregate supply for the market to clear:

$$\frac{W_t^{\text{o}} \lambda_t^{\text{o}} + W_t \lambda_t + W_t^* \lambda_t^*}{S_t} = Z$$

Using the definitions of relative wealth, we rewrite this market clearing equation as:

$$\frac{\varpi_t \lambda_t^{\text{o}} + \omega_t \lambda_t + (1 - \omega_t)\, \lambda_t^*}{S_t} = Z \tag{9}$$

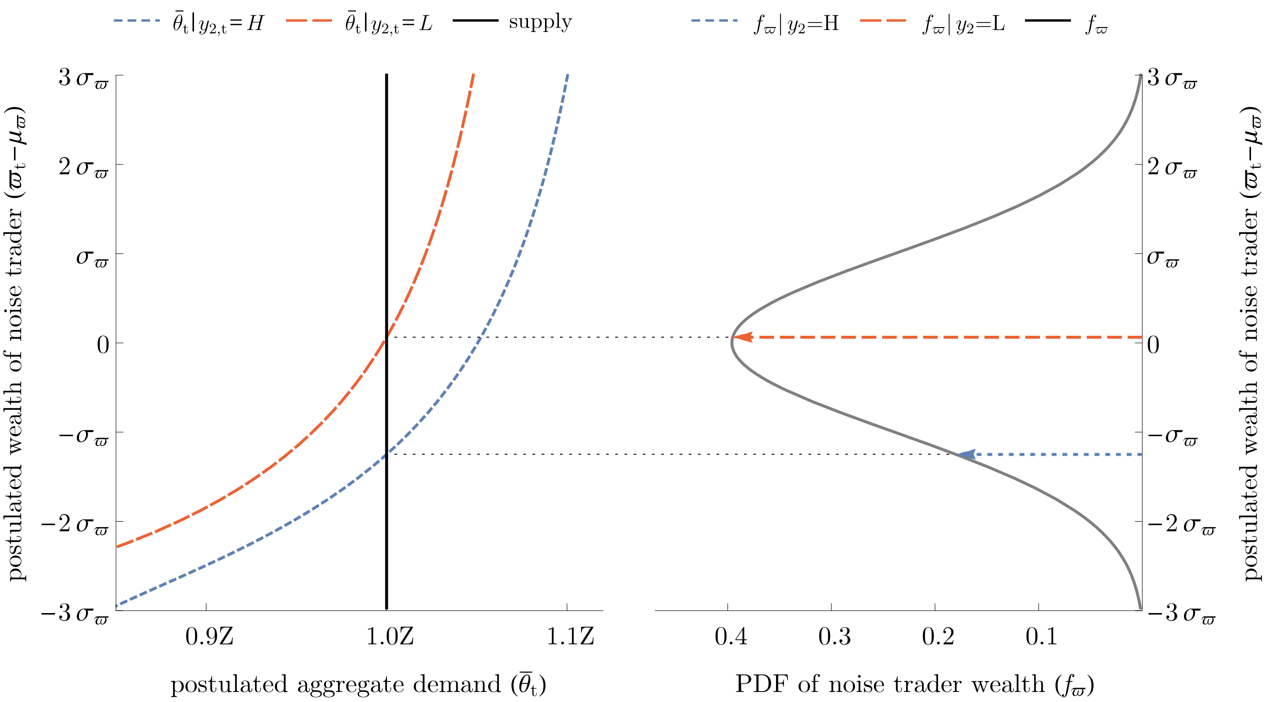

Figure 2: Leakage of private information through equilibrium prices. LEFT FIGURE (A): Upon observing the equilibrium stock price S_t, agent 1 computes aggregate demand for all *contemplated* realizations of the pair $\{\tilde{y}_t^*, \varpi_t\}$. The dashed lines represent these levels of aggregate demand for both realizations of $\tilde{y}_t^*$. For markets to clear, aggregate demand must equal aggregate supply. Only two the joint realizations $\left\{y^H, \varpi_t^H\right\}$ and $\left\{y^L, \varpi_t^L\right\}$ are consistent with this requirement. In the illustration above $\varpi_t^H = -1.2\sigma_\varpi$ and $\varpi_t^L = 0.1\sigma_\varpi$. RIGHT FIGURE (B): The relative likelihood between the two viable scenarios reveals information regarding y_t^*as stated in (11). The likelihood of each scenario is given by $f_\varpi\left(\tilde{\varpi}_t\right)\ell\left(\tilde{y}_t^*\right)$ where ℓ is given by (13) and $\tilde{\varpi}_t$ by (10). In our example, we can see that the likelihood of occurrence of $\tilde{\varpi}_t^L = 0.1\sigma_\varpi$ is around twice as high as those of $\tilde{\varpi}_t^H = -1.2\sigma_\varpi$. This makes it more likely that $y_t^* = y^L$ than that $y_t^* = y^H$ and thereby demonstrates how private information partially "leaks" through equilibrium prices.

3.3 Learning from prices

So far we have taken posterior probabilities π_t as exogenous. In this section, we show how agents learn from both private signals and equilibrium prices.

To gain basic intuition, it is useful to imagine a reference economy without noise traders. Then in each period only 4 possible equilibrium prices could be observed—namely those who clear the market for each realization of the pair $\{y_t, y_t^*\}$. Conditional on observing y_t there are two possible equilibrium prices, each consistent with one realization of y_t^*. Hence, $\{y_t, S_t\}$ is a sufficient statistic of y_t^* and prices are therefore *fully revealing*.

Now consider our full setup with noise traders. Since prices are affected not only affected by y_t^* but also by ϖ_t, there are multiple realizations of the pair $\{y_t^*, \varpi_t\}$ that are consistent with the observed price S_t. We write these *potential* or *contemplated* realizations with a tilde to distinguish them from actual (on-equilibrium) realizations. The two contemplated pairs $\{\tilde{y}_t^*, \tilde{\varpi}_t\}$—denoted by $\left\{y^H, \varpi_t^H\right\}$ and $\left\{y^L, \varpi_t^L\right\}$—are illustrated in by the two line intersections in Figure 2 (left). The noise trader wealth levels $\tilde{\varpi}_t \in \left\{\varpi_t^H, \varpi_t^L\right\}$ that are consistent with these two potential scenarios can be computed by inverting the market clearing condition:

$$\tilde{\varpi}_t = \frac{S_t Z - \omega_t \lambda_t - (1-\omega_t)\,\tilde{\lambda}_t^*}{\lambda_t^o} \tag{10}$$

Because we are uncertain which of the two potential scenario has occurred, $\{y_t, S_t\}$ is a not a sufficient statistic of y_t^* in the presence of noise traders. Nonetheless, the observation of S_t has helped us to has helped us focus our attention on only two possible scenarios $\left\{y^H, \varpi_t^H\right\}$ and $\left\{y^L, \varpi_t^L\right\}$. To learn about y_t^*, we need to determine the probability of each scenario occurring. Since the two unobservables are independent, this probability can be computed by taking the following likelihood ratio:

$$\mathbb{P}\left[\tilde{y}_t^* \,|y_t, S_t\right] = \frac{f_\varpi\left(\tilde{\varpi}_t\right)\ell\left(\tilde{y}_t^*\right)}{f_\varpi\left(\varpi_t^L\right)\ell\left(y^L\right) + f_\varpi\left(\varpi_t^H\right)\ell\left(y^H\right)} \qquad \forall \tilde{y}_t^*, \zeta_t \tag{11}$$

Where f_ϖ represents the PDF of the normally distributed wealth shock of the noise trader and ℓ the probability y_t^* conditional on y_t given by (13).[19] Since prices are functions of ζ_t, (11) must be computed for all realizations of ζ_t as well. It is important to highlight that only one realization of the contemplated pairs $\left\{y^H, \varpi_t^H\right\}$ and $\left\{y^L, \varpi_t^L\right\}$ has actually happened. The other pair still makes part of our fixed point problem, but is an off-equilibrium (hypothetical) realization only. Finally, with $\mathbb{P}\left[D_{t+1}\,|y_t, y_t^*\right]$ defined by (12), we use (11) and to compute posterior probability π_t:

$$\pi_t = \mathbb{P}\left[D_{t+1}\,|\{y_t, S_t\}\right] = \sum_{\tilde{y}_t^*} \mathbb{P}\left[D_{t+1}\,|y_t, \tilde{y}_t^*\right]\mathbb{P}\left[\tilde{y}_t^*\,|y_t, S_t\right] \qquad \forall D_{t+1}, \zeta_t \tag{14}$$

[19]Since y_t and y_t^* are separate draws from the Bernouilli distribution with probabilities ρ_t and ρ_t^*, the joint distribution of $\{y_t, y_t^*, D_{t+1}\}$ is given by:

$$\mathbb{P}\left[y_t, y_t^*, D_{t+1}\right] = \begin{cases} \frac{1}{2}\rho_t\rho_t^* & y_t^* = y_t = D_{t+1} \\ \frac{1}{2}\left(1-\rho_t\right)\rho_t^* & y_t^* \neq y_t = D_{t+1} \\ \frac{1}{2}\rho_t\left(1-\rho_t^*\right) & y_t \neq y_t^* = D_{t+1} \\ \frac{1}{2}\left(1-\rho_t\right)\left(1-\rho_t^*\right) & y_t^* = y_t \neq D_{t+1} \end{cases}$$

Using Bayes' law, we can compute the probability of D_{t+1} given y_t and y_t^*:

$$\mathbb{P}\left[D_{t+1}\,|y_t, y_t^*\right] = \frac{\mathbb{P}\left[y_t, y_t^*, D_{t+1}\right]}{\sum_{D_{t+1}}\mathbb{P}\left[y_t, y_t^*, D_{t+1}\right]} \tag{12}$$

Similarly, we can compute the probability of y_t^* given y_t :

$$\ell\left(y_t^*\right) = \mathbb{P}\left[y_t^*\,|y_t\right] = \frac{\sum_{D_{t+1}}\mathbb{P}\left[y_t, y_t^*, D_{t+1}\right]}{\sum_{D_{t+1}}\sum_{y_t^*}\mathbb{P}\left[y_t, y_t^*, D_{t+1}\right]} \tag{13}$$

3.4 System of equations

We next summarize the system of equations that must be solved at each period and each value of ω_t. In order to solve the economy specified above, we approximate the f_ϖ over grid with N points. The accuracy of this discretization can be made arbitrary precise subject to computational power availability. It remains now to solve a large fixed-point problem at each point in time.

Once the demand of noise traders (3) and the budget constraints (3) have been substituted, the remaining system consists of $16N$ first-order conditions with respect to (informed trader) portfolio holdings (4),[20] $16N$ consumption optimality conditions (5), $4N$ market clearing equations (9), $16N$ learning equations (14), and 2 information acquisition first-order conditions (6).

Recall that there are $4N$ possible (on-equilibrium) realizations of ζ_t. There are thus $16N$ unknowns for λ_t and λ_t^*, $16N$ unknowns for c_t and c_t^*, $16N$ unknowns for π_t and π_t^*, $4N$ unknowns for S_t and 2 remaining unknowns ρ_t and ρ_t^*. In total, this a system yields $52N + 2$ equations and unknows.

3.5 Infinite horizon problem

We next show to how to extend the Breugem and Buss [2018] algoritm to a dynamic setting. By eliminating total wealth in the system of equations, we have significantly simplified the numerical analysis of the economy. There is only one state variable ω_t and there are two state functions V_t and V_t' that are functions of both ω_t and time. We compute V_t and V_t' by (i) solving the system of equations and then (ii) evaluating (1) at every value of ω_t. Since V_t and V_t' depend on V_{t+1} and V_{t+1}', we need to start solving at the final node of the model, and solve the system backwards.

In order to determine the "steady" distribution of wealth in our economy, we need to solve an infinite horizon economy ($T \to \infty$). This is a nontrivial task since V_t and V_t' are functions of time (and horizon). We solve this issue using both an existing method and a new method that is innovative to the literature. Both methods have their own computational benefits and drawback, but yield identical results.[21]

The first method is recursive and is adapted from Buss and Dumas [2016]. We start at the last node, at which V_T and V_T' are simply given by the imposed utility function. Working backwards, V_t and V_t'

[20] Specifically, there are 2 traders, 4 on-equilibrium (combined) signal realizations and 2 contemplated (private) signal realizations, yielding $2 \times 4 \times 2 = 16$ equations per realization of ϖ.

[21] As always is the case with numerical solutions, more accurate results are achieved when the quality of the approximation is higher.

are solved over a grid Ω of M values of ω_t. Since $\beta < 1$, the degree to which future utility consumption contributes to the value function is lower in more distant periods. Therefore if we work backwards "far enough", V_t (and therefore V_t') do not change anymore. We stop solving when the two state functions, V_t and V_t' are close enough to V_{t+1} and V_{t+1}'. This is the case as the maximum deviation (in terms of ω) is below a certain treshold ϵ:[22]

$$\sum_{\omega \in \Omega} \left(\frac{V_{t+1}[\omega] - V_t[\omega]}{V_t[\omega]} \right)^2 < \epsilon \tag{15}$$

Condition (15) simply states that the value function does not change anymore if we iterate further backwards. Of course, ϵ must be small enough and the left hand side of (15) must be converging to zero. Convergence will be faster when β is lower.

A drawback of the above method is that potentially many backwards iterations could be needed before the state functions V_t and V_t' are sufficiently stable. Numerical mistakes, for example due to interpolation, that are innocent in a static framework could be amplified dynamically. In addition, the speed of convergence will be a function of the model parameters (such as β) which makes it difficult to predict how long the algorithm takes to execute and therefore what the corresponding optimal numerical precision that balances speed and accuracy should be.

The second method, which to our knowledge is new to the literature, gets rid of these problems by solving the infinite horizon problem in just one step. We describe the functions V_t, V_t', V_{t+1} and V_{t+1}' represent the interpolated values of series of M function values over $\omega \in \Omega$. We denote these function values by $v_{\omega,t}$, $v_{\omega,t}'$, $v_{\omega,t+1}$ and $v_{\omega,t+1}'$, yielding $4M$ variables in total.

We add these $4M$ variables are not obtained recursively, but instead added as unknowns to a larger system of equations. This larger system consists of (i) M versions of the "baseline" system described in section 3.4 at every gridpoint ω_t, (ii) $2M$ computations $v_{\omega,t}$ and $v_{\omega,t}'$ for a given $v_{\omega,t+1}$ and $v_{\omega,t+1}'$ using and (iii) $2M$ infinite horizon conditions that imposes a time-invariance value functions:

$$\begin{aligned} v_{\omega,t} &= v_{\omega,t+1} \qquad \forall \omega \in \Omega \\ v_{\omega,t}' &= v_{\omega,t+1}' \qquad \forall \omega \in \Omega \end{aligned}$$

The system to solve consists of $(52N + 4)\, M$ equations and unknowns. A great benefit is that this system needs to be solved only *once*, to fully characterize the infinite horizon model. A drawback

[22] A similar condition must hold for V_t'

is of this method is obviously the massive requirement on computational power.[23] The desktop PC currently at our disposal is capable of solving this problems for lower values of M and N only. We nonetheless present this new solution method since it less subjective to amplification of numerical mistakes as the recursive method. We also believe that the implementation of the method can be done for higher values M and N in the near future if computing power continues to increase exponentially.[24,25]

4 Information acquisition

The remainder of this paper presents our findings. In sections 4-6, we exclusively present results for $T = 1$. We commence the step-by-step analysis of the solution of our fixed point problem by investigating the equilibrium information acquisition decision of a single agent. This is our first result:

Result 1. *Wealthier agents invest more in private information*

This finding, illustrated in 3a, is robust to various parameterizations. To understand Result 1, we highlight that information yields returns to scale, since it can be applied to an entire portfolio regardless of its size. In a setting without information acquisition, a CRRA-agent would allocate a *fraction* of her portfolio to the risky assets. She chooses to do so since her *absolute* risk aversion is proportional to her wealth. When she is allowed to buy information, she is increasingly willing to do so whenever her risky portfolio is larger. A wealthier CRRA agent therefore buys more private information.

An investor with CARA preferences would not demand more information in case he is richer. Indeed, absent information purchase, he would simply invest a constant dollar-amount in risky securities, resulting in a wealth-insensitive demand for information. A reproduction of Figure 3 would be a collection of horizontal lines. This is clearly a less realistic assumption as it would imply that Warren Buffet would invest the same resources on information as the author of this paper.

We believe that Result 1 holds for both private and professional active investors of we abstract away from institutional frictions. Since the compensation of fund managers is typically a fraction of their assets under management (AUM), managing a larger fund entitles the manager to a higher wage.

[23]Suppose $M = 20$ and $N = 25$. Then the system consists 26080 equations and unknowns. A Newton algoritm to solve this system would need to work with a Jacobian of $26080^2 \approx 0.68$ billion variables. Although many entries of the jacobian are zero, the memory pressure is obviously still huge.

[24]E.g., following the so far consistentMoore [1965]'s law predicting a doubling of computing power every year.

[25]The biggest task of the numerical algoritm is to invert the huge Jacobian (or to find its LU-Decomposition) is a set of computations that can be executed (to a large degree) in parallel. Recent developments in Graphical processing Unit (GPU) programming might therefore render our "global" method more feasible in the near future.

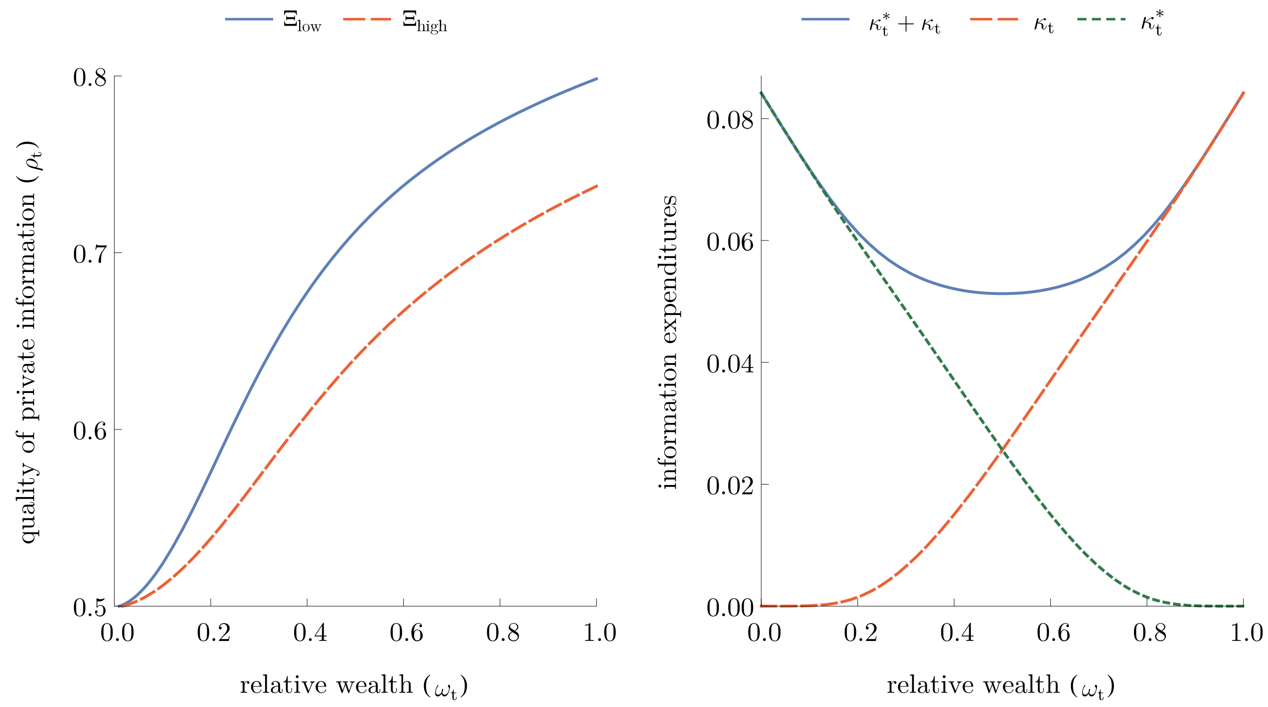

Figure 3: Acquisition of private information. LEFT FIGURE (A): Richer agents acquire more private information. The convexity of the information cost function ensures that perfect foresight ($\rho_t = 1$) is impossible to attain in the presence of costly information. In general, a higher the multiplicative constant Ξ results in a lower quality of private information, and decreases inequality in information allocation. RIGHT FIGURE (B): Expenditures on information are an increasing and convex function of an agent's (relative wealth). Therefore, the aggregate dollar amount spent on information is larger in economies with unequal capital allocation.

Managers with CRRA preferences over their salaries are therefore willing to spend more resources on performance-enhancing methods (e.g., by hiring experts, visit conferences, subscribe to proprietary news channels) when they manage bigger funds.

Note the focus of this paper to relate the *distribution of wealth* to information allocation and asset prices. Since all key variables in our setup scale with aggregate wealth, only capital *inequality* matters here. We thereby complement Peress [2003], who fixed the distribution of wealth and focuses on parallel shifts instead.[26] We find that inequality in wealth matters not only for individual information allocation but also for aggregate spendings:

Result 2. *Expenditures on skill are convex in an agent's (relative) wealth. Therefore, total spendings on skill are larger in economies with larger capital inequality.*

Figure 3b displays the result. Investors spent disproportionally more wealth on private information when they are wealthier. Total spendings are therefore increasing in capital inequality, which is a new result to the literature.

[26]Result 1 simply confirms Peress [2003] beyond his small-risk approximation.

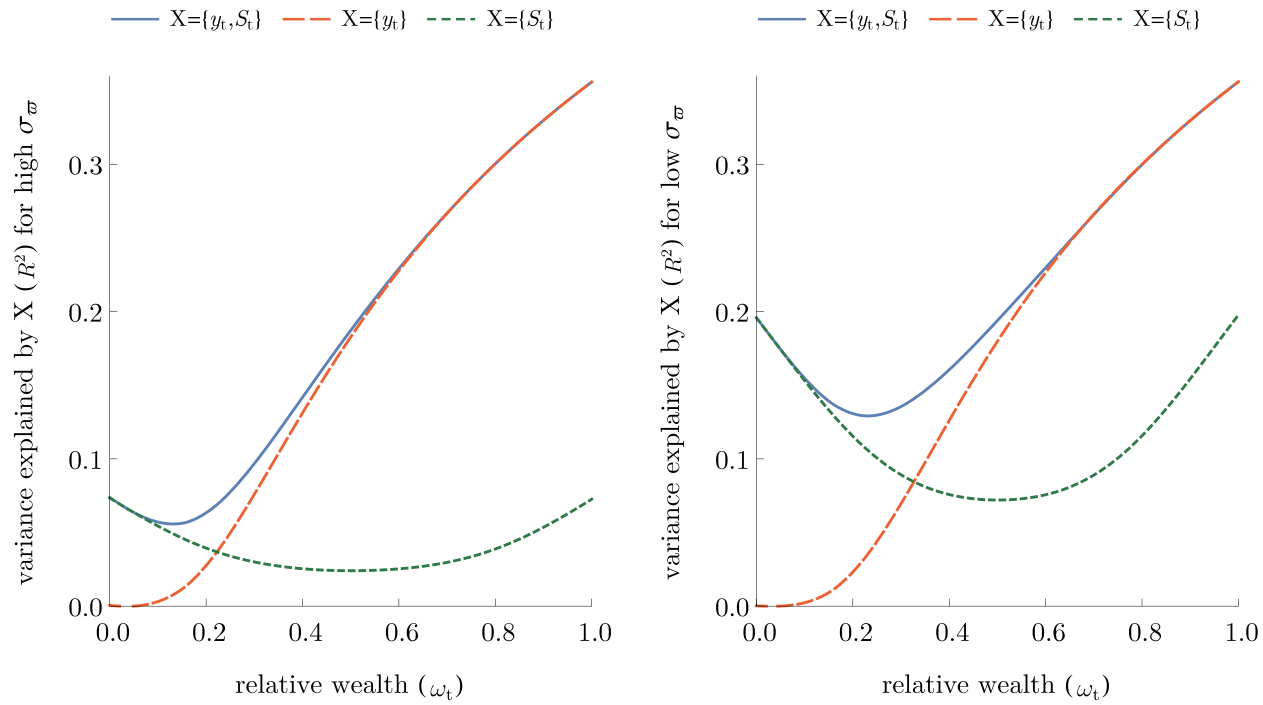

Figure 4: Information aggregation. LEFT FIGURE (A): The solid line presents the posterior information set. The information set can be non-increasing for low values of relative wealth. This is possible since at the extremes of the wealth distribution, the richer agent's information sets is at its peak and leaked extensively through equilibrium prices (see thinner dashed line). The size of the free riding effect (difference between solid and thicker dashed line) is decreasing in relative wealth. RIGHT FIGURE (B): Identical setting as left panel but with low volatility of noise trader's wealth, which increases simplifies the learning process from public information and thereby increasing the information content of equilibrium prices.

As a final note we point out that Result 2 does not make any welfare statements. While the inequality statements made in this section could be suggestive of generating disutility for the ex-ante (identical) agent, we have not yet considered information aggregation nor did we endogenize the distribution of wealth. Indeed, as in any REE framework, private information can leak through equilibrium prices, which can be beneficial for agents with inferior private information. This effect is studied in the next section.

5 Information aggregation and free-riding

Consistent with the REE literature, investors in our economy (except noise traders) are rational, perfectly understand the economy and thereby know that the observed equilibrium price can only be consistent with a certain realization of signals of the other agent. The details of the learning technique were described in detail in Section 3.3 and the current section present our main findings, which are displayed in Figure 4:

Result 3. *Prices reflect more private information when wealth inequality is higher*

This finding is consistent with 2 and relates back to the returns-to scale mechanism discussed in the previous section. The wealthier agent allocates disproportionally more resources to information which contributes to a higher informativess of prices in unequal economies. Even if we kept the total level of information in the economy fixed, it remains that the wealthier agent acts more on her information by employing more aggressive trading strategies, which induce prices to "move" even more.

Agents with low wealth benefit most from this "informational free-riding" effect. The fraction of an agents posterior information that has been obtained for free (by using the public signal), is a monotonically decreasing function from one at $\omega_t = 0$ to zero at $\omega_t = 1$. Therefore, richer agents do not only acquire information for themselves, but also "work" for the community of rational traders.

The increase in price-informativeness at the edges of the wealth distribution ensure that in unequal economies, investors can to a larger extend free-ride on private information of others. This means that the quality of posterior information is *not* monotonically increasing in the level of relative wealth! Hence, all else equal, very poor agents prefer to be in an economy with a richer agent, who buys information poor agents can learn from for free.

Figure 4 shows how the size of the informational free-riding effect is affected by the level of noise, represented by the volatility of the noise trader's wealth. First, we observe that in our price-taking economy, the level of noise does not affect much the decision to buy private information.[27] By decreasing the level of noise, the fraction of variance attributed to private signals (compared to noise) is larger, which increases price informativeness. As a consequence, (i) informational differences decrease and (ii) average level of posterior information when the level of noise is larger.

6 Portfolio performance, inequality and growth

We next investigate implications for portfolio performance. As discussed in the previous section, two key effects are driving forces behind the following finding:

Result 4. *Wealthier investors yield higher returns on their portfolios. The difference in portfolio performance has an inverted U-shape with respect to capital inequality.*

[27] This mostly because we assume agents are price taking and would be very different in case our agents were strategic a la Kyle [1985].

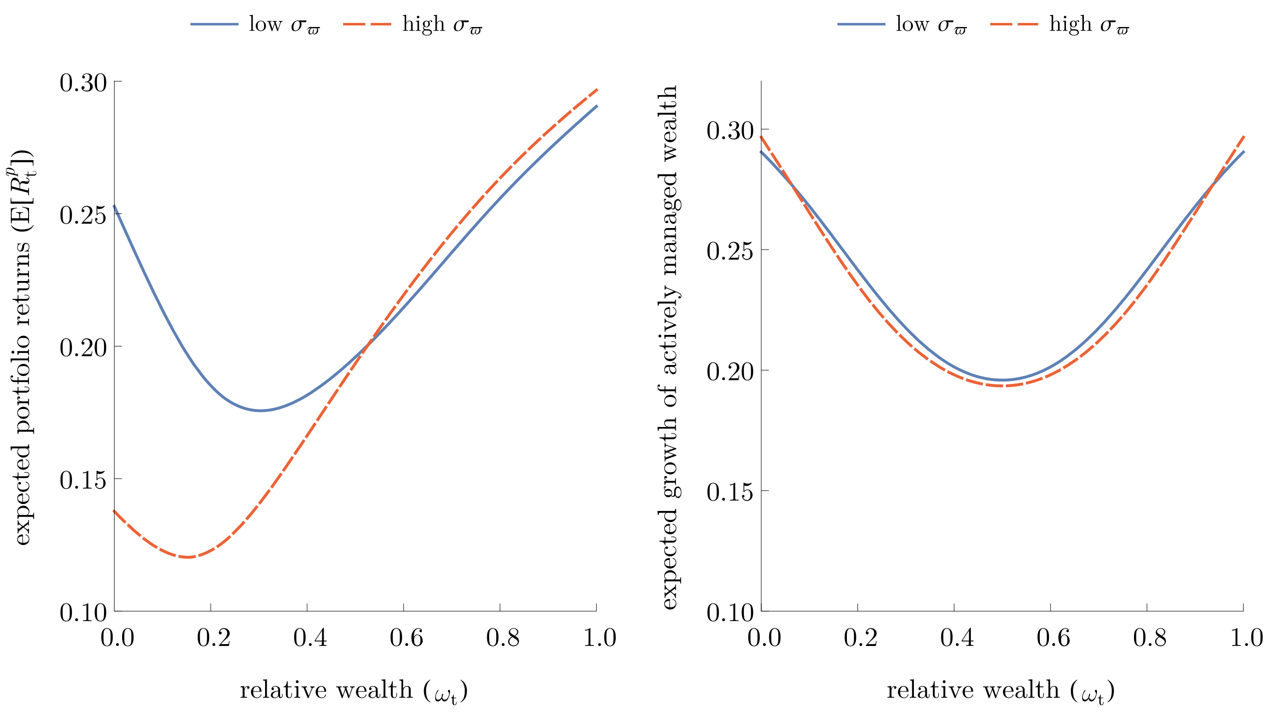

Figure 5: Portfolio performance and industry growth. LEFT FIGURE (A): Expected portfolio performance as a function of relative wealth. The difference in portfolio performance has an inverted U-shape with respect to capital inequality the presence of two opposing forces: the informational free-riding effect and the information return to scale effect. RIGHT FIGURE (B): Expected growth of the actively managed portfolio industry (wealth of informed investors) is higher when capital inequality is high.

On the one hand, due to informational returns to scale, richer agents acquire more private information and therefore yield higher returns on their portfolio. By this mechanism, portfolio performance should be monotonically increasing in (relative) wealth. On the other hand, due to informational free-riding, investors yield higher expected returns in the presence of other (much) wealthier speculators. This effect was discussed in the previous section (Result 1): When capital is very unequally distributed, prices reflect more private information and the least wealthy agent free-rides on the rich agent's private information expenditure. At very unequal levels of capital, the former effect can be dominated by the latter effect, which generates the inverted U-shape relation between portfolio performance and wealth inequality.

Figure 5a plots portfolio performance as a function of relative wealth for different levels of noise. For both levels of noise, we see that portfolio performance is increasing in relative wealth when capital inequality is low ($\omega_t \approx \frac{1}{2}$), but decreasing when capital inequality is high (ω_t closer to either 0 or 1). The latter effect is stronger when there is less noise in the economy. Not only does this arise because of the improved performance of the poor agent, but also due to the weaker performance of the rich agent in the absence of noise. Indeed, since both speculators have a common source of profits (noise traders), the better performance of the poor agent goes at the cost of the performance of the rich agent. We find, however, that this is not a zero-sum game between the two speculators:

Result 5. *Average (industry) expected performance of informed traders increases in capital inequality.*

Figure 5b shows this result, which can be understood by looking at Figure 4. For high levels of capital inequality, the (weighted average) quality of posterior information is higher. Therefore, the (weighted) average expected performance is higher in economies with unequal levels of wealth. If we interpret our two endogenously informed agents as representing the active portfolio management industry, we then see that the industry grows fastest when capital is unequally distributed.

7 Equilibrium dynamics of wealth and skill distribution

So far, our analysis has focused on the static version of the model in order to highlight the key theoretical mechanism. Our next step is to endogenize the distribution of wealth in a dynamic model. We do so by solving a dynamic optimization problem for T large enough, so that a stationary distribution of relative wealth is found. Our first finding is the following:

Result 6. *When the level of noise trading is sufficient low, there exists an interior equilibrium distribution of wealth* $\omega_t \in (0, 1)$.[28]

We gain highlight our main intuition in Figure 6a. For any level of relative wealth (more or less) on *the interval* $\omega_t \in (0.1, 0.9)$, capital inequality is expected to grow. In this region, the informational returns to scale effect dominates: richer agents purchase more information, and get comparatively even richer. In more extreme ranges of the wealth distribution, when $\omega_t \in (0, 0.1) \cup (0.9, 1)$, the free-riding effect dominates, which results in an expected decrease in capital inequality. At the *exact points*

[28] So far we have documented that richer agents obtain, in expectation, superior portfolio returns. Even in the low-noise version of figure 5a, we find that the richer agent ourperformes his poorer counterpart for *all levels of relative wealth.* To understand that this result does not contradict 6, first note that:

$$\mathbb{E}_t\left[\omega_{t+1}\right] = \mathbb{E}_t\left[\frac{W_{t+1}}{W_{t+1}+W^*_{t+1}}\right] \neq \frac{\mathbb{E}_t\left[W_{t+1}\right]}{\mathbb{E}_t\left[W_{t+1}\right]+\mathbb{E}_t\left[W^*_{t+1}\right]} = \frac{\omega_t \mathbb{E}_t\left[R^P_{t+1}\right]}{\omega_t \mathbb{E}_t\left[R^P_{t+1}\right]+\omega^*_t \mathbb{E}_t\left[R^{P*}_{t+1}\right]} \tag{16}$$

In words, the expected fraction of future wealth does not equal to the ratio of (weighted) expected portfolio performance. This is because when agents have different levels of risk aversion, that in our model defacto arises due to heterogeneous posterior information sets, the risk averse (=poor) agent wishes a smoother consumption and has a comparatively fraction of wealth in bad states of nature than in good states of nature. bad states of nature, however, contribute more towards the computation of the left hand side of (16) than good states do. Consequently, the wealth share of the poor agent could grow in expetation at the extreme ends of the distribution.

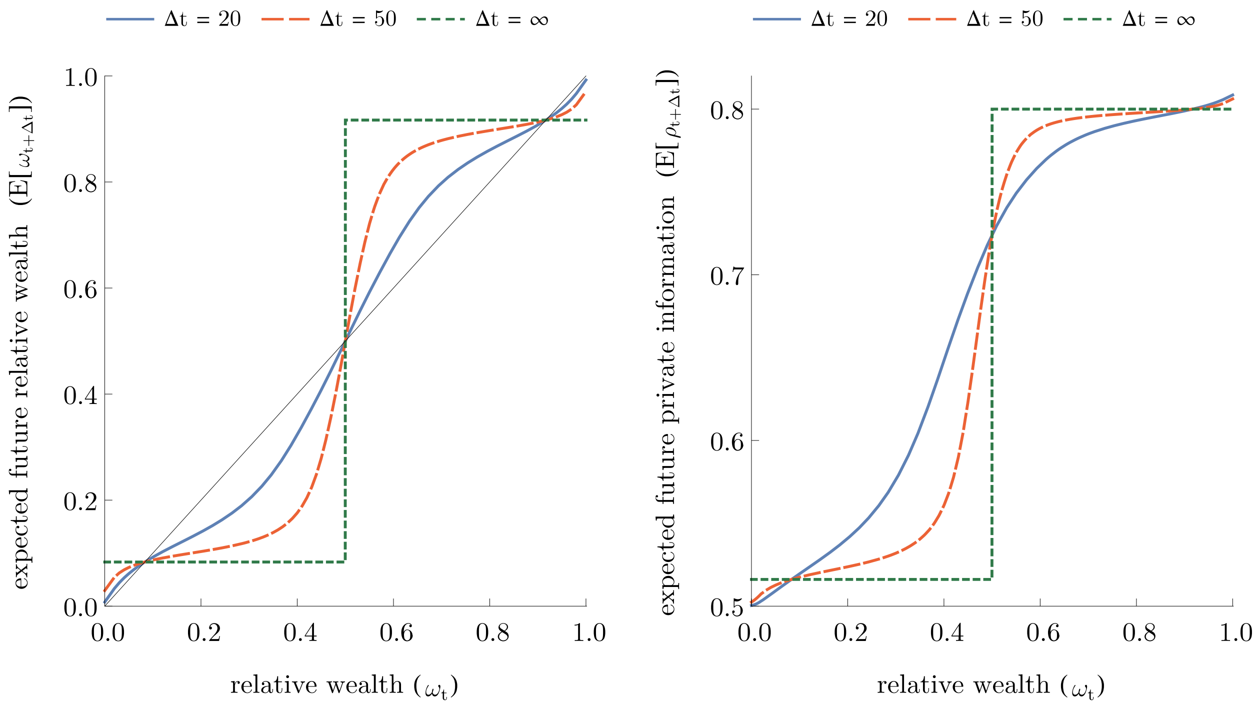

Figure 6: Equilibrium dynamics of the distribution of wealth. LEFT FIGURE (A): Expected change in relative wealth after various trading rounds. The dispersion of wealth expected to increase further for low levels of capital inequality, and decreasing for high values of capital inequality. Informational returns to scale generate a capital inequality-increasing force. The informational free-riding effect prevents wealth dispersion to grow indefinitely. The equilibrium dispersion of wealth is at $\{\hat{\omega}_t, \hat{\omega}_t^*\} = \{0.1, 0.9\}$. RIGHT FIGURE (B): Corresponding expected difference in distribution of private information after various trading rounds. The equilibrium dispersion of wealth is at $\{\hat{\rho}_t, \hat{\rho}_t^*\} = \{0.515, 0.805\}$. Both figures are generated under infinite horizon ($T \to \infty$).

$\hat{\omega}_t \in \{0.1, 0.9\}$, investors are expected to keep their relative wealth position and the two countervailing effects are balanced.

The finding that the stationary points $\hat{\omega}_t$ are not necessarily at the edges, indicates that the increasing returns to scale in active portfolio management are not, in general, strong enough to lead to a natural monopoly position. This result seems to be in line with empirical evidence found in **?**, who document a decreasing returns to scale for the mutual fund industry. Moreover, our finding is in line with Berk and Green [2004] bus also holds when fund flows are absent. Our finding reconciles the mutual fund literature following Berk and Green [2004] with the theoretical REE literature on information acquisition. Due to learning from prices, our result is also different than would be found in a (non-financial) industry equilibrium a la Romer [1990].

We next investigate the stationary distribution of wealth as a function of noise trading (market efficiency). The results are displayed in Figure 7a. For large values of noise trading (inefficient capital markets), the informational free-riding effect is weaker, and a more unequal distribution of wealth is

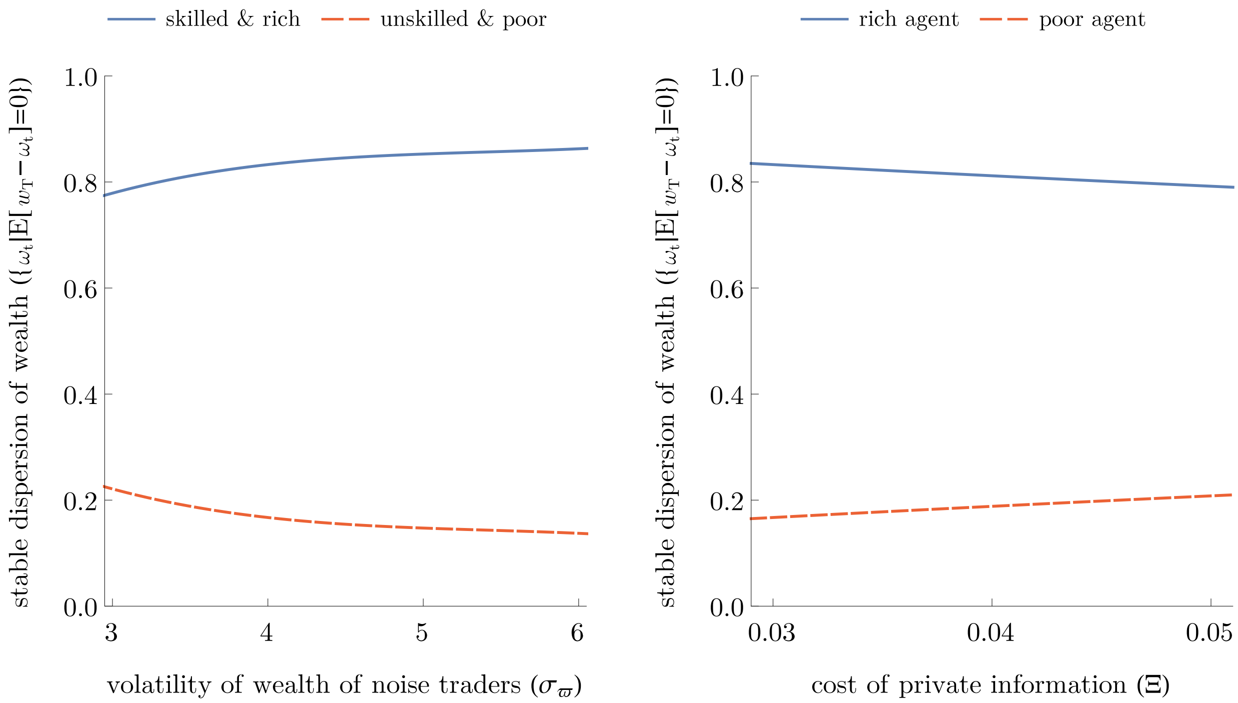

Figure 7: Characterization of the equilibrium dispersion in wealth. For a very large T, we compute the level of relative wealth $\hat{\omega}_t$ which is expected remain constant over time. LEFT FIGURE (A): higher level of noise, measured by the volatility of endowment of noise traders, diminishes price informativeness and therefore also the ability to free-ride on public information. Investors can now trade more aggressively without leaving an informational trail, which increases the equilibrium dispersion of wealth. RIGHT FIGURE (B): Higher information acquisition costs decrease the ability of the rich agent to grow even richer and reduce the equilibrium dispersion of wealth.

supported. When the market is sufficiently inefficient (not shown), the financial market resembles a standard industry a la Romer [1990], yielding a corner solution $\hat{\omega}_t \in \{0, 1\}$.

The opposite holds for an increase in information costs Ξ. A higher cost of information disables the richer agent from expanding his wealth compared to the poor agent. This decreases the equilibrium dispersion in wealth.[29]

8 Asset pricing and wealth inequality

We finally investigate the asset pricing implications of our model with endogenous skill. To highlight the impact of informational differences, we first demonstrate a finding of a static model:

Result 7. *Expected returns and volatilities of returns on stocks are lower (asset prices are lower) in unequal economies*

[29]Figure 7 is generates for $T = 10$, which is a preliminary result only. We are currently working on a model with $T \to \infty$

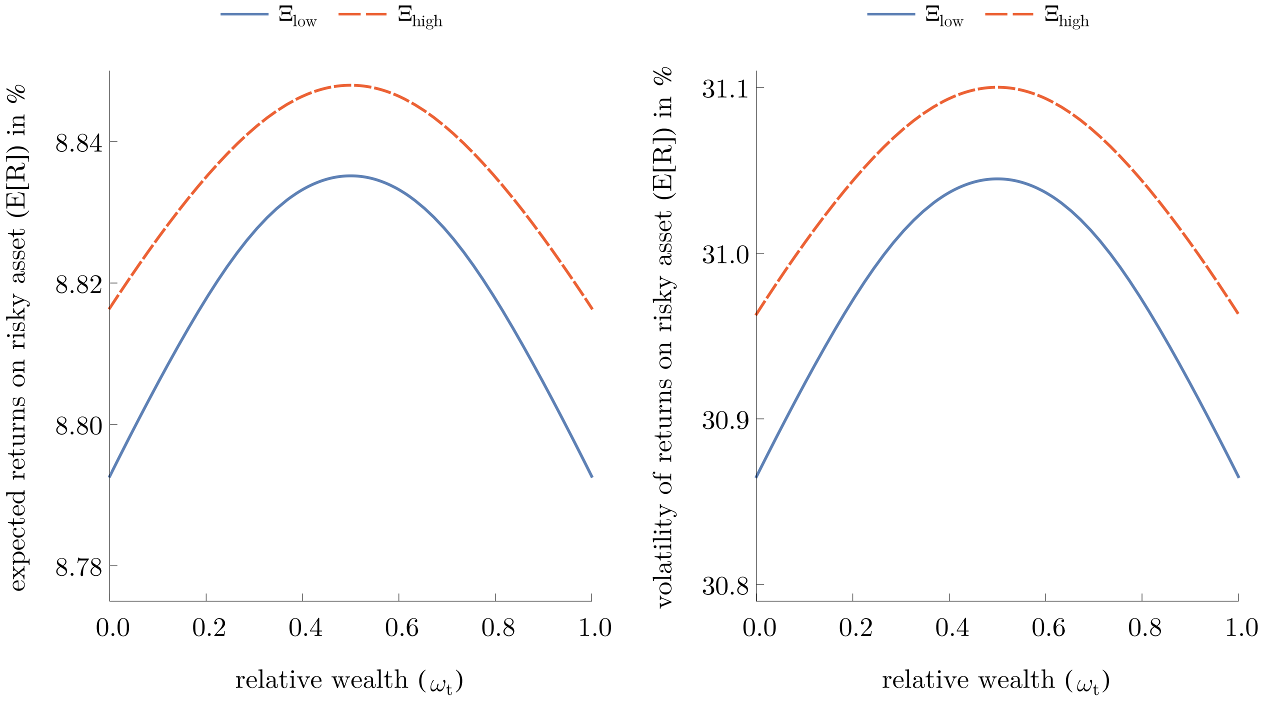

Figure 8: Asset pricing and capital inequality. LEFT FIGURE (A): expected returns as a function of wealth inequality. When wealth is unequally allocated, prices reflect more information. This yields a higher speculative demand for information and increases expected returns in equilibrium. RIGHT FIGURE (B): High dispersion in capital results in more informative prices leading in lower residual uncertainty and therefore smaller volatility of returns on risky investments.

This finding can best be understood by recalling Result 3. When capital inequality is large, prices reflect more private information. Or, the weighted average posterior information set of the two speculators is larger. As a consequence, trading in the risky asset is perceived to be less risky when the dispersion in wealth is high, yielding lower volatility of returns. Given the lower (perceived) volatility of investment, agents should get a lower expected returns on their investment.

This finding contributes to the literature on wealth inequality and asset pricing. Gollier [2001] shows that capital inequality can increase the equilty premium if absolute risk aversion is concave in wealth. Our setup does not impose such preferences, but the defacto attitude of an investor towards risk arises endogenously in the presence of costly (and free) information. Pastor and Veronesi [2016] focus on income rather than wealth inequality in a framework without learning and focus on tax implications rather than information aggregation.

9 Empirical implications

This section discusses empirical implications of our model. We contribute to the literature by linking information allocation to the distribution of wealth, and thereby connecting an unobservable variable to an observable variable. By doing so we bring the rational expectations equilibrium literature a step closer to the econometrician. Finally, we provide examples of such testable hypothesis that are typical to our model.

9.1 Instrumental variables for (unobservable) private information

Despite being rich in theoretical results, the rational expectations equilibrium (REE) literature following Grossman and Stiglitz [1980] is full of implications that are hard to test empirically. First, noise trading is key ingredient to most REE models, but is empirically hard to quantify. Second, predictions of REE models are particularly sensitive to distributional assumptions. Specifically, the dual role of equilibrium prices (market clearing and information aggregation) ensure that deviations from the reference framework can have large and sometimes even reverting effects.[30] Third, since there are only a certain number of degrees of freedom in a traditional REE model, many papers predict a similar transformation rule between input and output variables. The theoretical mechanism across papers generating a certain transformation rule could differ greatly. However, the econometrician can often not pin down the exact mechanism, since many (intermediate) propositions of the model are linked to unobservable variables (such as information), and thereby not testable.

A theoretical mechanism could be pinned down more easily if intermediate propositions are testable. This is where our paper contributes. By using CRRA preferences, we link information acquisition to the distribution of wealth in the economy. A novel contribution of our paper is that we keep aggregate wealth untouched[31] and only focus on capital inequality.

First, we contribute to the testability of Pastor et al. [2014] . Specifically, Pastor et al. [2014] explain the reduction of the "scope" for active portfolio management to a reduction of noise trading. Since noise traders are a source of trading profit to informed speculators, any standard rational expectations

[30] Several papers, including Bernardo and Judd [2000] and more recently Breon-Drish [2015] show the vulnerability of the traditional CARA/normal framework to distributional assumptions. What the "correct" distribution of input variables should remains an empirical question. Recently, Peress and Schmidt [2015] constibute to this issue by estimating a "realistic process" for noise trading.

[31] In fact, by assumption, we rule out any affects arising from shifts in aggregate wealth.

equilibrium (REE) model similar to Grossman and Stiglitz [1980] would produce this results, regardless of preferences.

If a reduction in noise trading is really the cause of a decrease in scope for the active portfolio management industry, the some additional implications must hold as well. Specifically, by we predict the following:

Hypothesis 8. *A reduction of "noise" does not only reduce the performance of the asset management industry, but also reduces the dispersion of skill and wealth.*

According to our model, a reduction in noise trading should also decrease capital inequality and dispersion in "skill". Indeed, since wealth and skill dispersion are endogenous in our setup, they should be affected by noise trading accordingly. It is important to measure wealth and skill for the asset management industry, rather than for households when performing the empirical test.

Of course, our setup ignores several mechanisms that are key to the asset management industry. Our model could be extended by modeling fund flows. In this setup, which would get a similar flavor as Berk and Green [2004], fund flows ensure informed portfolio managers do not outperform each other. We can still test Hypothesis 8 since funds flow enlarge the distribution of wealth even further. Whether or not fund flows amplify or dampen our core mechanism is a question for future research.

9.2 Can one measure the "skill" of active managers?

The existing literature on mutual funds assumes that managerial "skill" behaves like a built-in feature of an investor. Until recently, there was a consensus belief that managers are not skilled as their alpha was on average negative. Recent literature[32] has pointed out, however, that skill should not be measured with alpha, but rather with dollar value added.

This intuition is very clearly explained in Berk and van Binsbergen [2013]: Suppose there are two (open-end) fund managers, A and B. With identical AUM, manager A is able to obtain 20% returns on her portfolio, where B only can obtain 10% returns. If funds are endogenous and investors are rational, then the market will allocate more wealth to investor A until the expected rate of return[33] for A and B is identical. For this reason, one cannot look at alpha to measure manager skill, but one

[32](e.g., Berk and Green [2004], **?**)

[33]When investors are risk averse, expected utility from investing (instead of expected returns) should be identical.

should look at AUM instead. More precisely, if managers can set their fees and therefore gauge fund inflows, the dollar value added (DVA) would be the appropriate measure for skill.

In the above example, managers with identical skill would always yield similar DVA. In our setup, this would not be the case. Managers with identical skill could endogenously differ in AUM which determines their ability to enhance their portfolio returns by acquiring private information. In the long run, managers with identical skill could yield vastly different portfolio returns.

The econometrician can only measure the combination of personal talent and acquired private information by looking at DVA. The latter, however, is driven by lucky performance in the past which makes it difficult to pin down talent on its own. Put if differently, the smartest manager might be stuck at a small fund due to unlucky initial draws. In general, we claim that the (dynamic) endogeneity between luck and skill makes it harder to investigate (i) whether or not portfolio managers are inherently smart and (ii) which ones are the smartest.

10 Discussion

We study the distribution of wealth and skill in a model of active portfolio management. Our model extends Verrecchia [1982], and Breugem and Buss [2018] to a **dynamic setting** with heterogeneous investors with CRRA preferences. Two opposing forces are determine the equilibrium distribution of capital: One the one hand, information can be applied to any portfolio size, which generates increasing returns to scale. On the other hand, large speculative trades leave an informational footprint by affecting equilibrium prices. We determine asset pricing, skill dispersion and information aggregation implications at the equilibrium distribution of wealth.

A straightforward extension of our model would be the incorporation of endogenous flows. This will speak more closely to the situation of active portfolio management in general as it describes the typical problem of an open-end fund. This extension adds a requirement that the expected utility for the (small) investor by investing in the funds should be identical and thereby further endogenizes the distribution of wealth.

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