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REGULAR ARTICLE

Factor vector autoregressive estimation: a new approach

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Abstract In this paper a new approach to factor vector autoregressive estimation, based on Stock and Watson (Implications of dynamic factor models for VAR analysis, NBER Working Paper, no. 11467, 2005), is introduced. In addition to sharing all the relevant features of the Stock–Watson approach, in its static formulation, the proposed method has the advantage of allowing for a more clear-cut interpretation of the global factors, as well as for the identification of all idiosyncratic shocks. An application to large-scale macroeconometric modelling is also provided.

Keywords Factor vector autoregressive models · Large-scale macroeconometric models

JEL Classification C32

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1 Introduction

Factor vector autoregressive (FVAR) models are a recent important novelty in the literature, allowing to easily handle the estimation of large-scale dynamic econometric models in the convenient vector autoregressive (VAR) framework. Seminal contributions can be traced back to the works of Giannone et al. (2002), Bernanke and Boivin (2003), Favero and Marcellino (2005), Favero et al. (2005), Bernanke et al. (2005), Pesaran et al. (2004), and Stock and Watson (2005). A common element in all the above studies is the intuition that the information contained in very large data sets of economic variables can be summarized by a small number of factors, which can then be employed to augment vector autoregressive systems, in order to allow for a better description of economic dynamics in macroeconometric models.

Yet, some differences can also be noted among the various approaches. In fact, while in Bernanke and Boivin (2003), Giannone et al. (2002), Favero and Marcellino (2005), Favero et al. (2005), and Bernanke et al. (2005), the framework of the analysis is still a small-scale macroeconometric model, where the additional factors are introduced to alleviate the problem of omitted variables, the contributions of Pesaran et al. (2004) and Stock and Watson (2005) aim at large-scale or global macroeconometric modelling. Important differences can also be found in terms of estimation methods. For instance, in all the above papers apart from Stock and Watson (2005), estimation is based on a two-step approach, where first the economic factors are estimated either by the use of principal components analysis (PCA) as in Stock and Watson (1998), or by means of frequency domain principal components as in Forni et al. (2000), or as cross-sectional averages of observed variables as in Pesaran et al. (2004), and then included as exogenous variables in VAR-X models. Yet, in the above papers also one-step estimation, based on the Kalman filter approach or Bayesian likelihood methods and Gibbs sampling, are discussed. Differently, Stock and Watson (2005) have proposed an iterated two-step approach, still based on the use of PCA, which however allows to recover the efficiency of the single-step estimator. In the light of the recent results of Bai (2002, 2003) and Bai and Ng (2004), which have justified the use of PCA also for the case of strongly persistent processes, the iterated two-step approach appears to be optimal, given its asymptotic properties and simplicity of implementation.

In this paper, a new approach to FVAR modelling and estimation is proposed. The proposed approach is an extension of Stock and Watson (2005), allowing for a more straightforward interpretation of the global factors and for the identification of all shocks, both global and idiosyncratic. Moreover, differently from the FVAR approach of Favero et al. (2005), Giannone et al. (2002), and Bernanke et al. (2005), the proposed method has the advantage of using an iterated procedure in estimation, recovering full efficiency asymptotically, allowing, as in Stock and Watson (2005), the imposition of appropriate restrictions concerning the lack of Granger causality of the variable versus the factors as well. In addition, relatively to the approach employed by Pesaran et al. (2004), as in Stock and Watson (2005), modelling is not carried out by considering each block of relevant variables at the time, on the basis of long-run forcing, but all the variables are treated as endogenous at the outset. Finally, in the proposed framework the unknown factors can be interpreted as global factors, while in Pesaran et al. (2004)

the interpretation is less straightforward, also due to the way the weighting for the factors is chosen.

After this introduction, the paper is organized as follows. In Sect. 2 the econometric methodology is presented, while in Sect. 3 an application to the estimation of a large-scale macroeconometric model for the G-7 countries is provided. Conclusions are drawn in Sect. 4.

2 Econometric methodology

Following Stock and Watson (2005), consider the following factor model

$$X_t = \Lambda F_t + D(L)X_{t-1} + v_t \tag{1}$$

$$F_t = \Phi(L)F_{t-1} + \eta_t, \tag{2}$$

where X_t is a n-variate vector of variables of interest, F_t is a r-variate vector of global factors, v_t is a n-variate vector of idiosyncratic i.i.d. shocks, η_t is a r-variate vector of common or global i.i.d. shocks, $E\left[\eta_t v_{is}\right] = 0$ all i, t, s, Λ is a $n \times r$ matrix of loadings, and $D(L)$, $\Phi(L)$ are matrices of polynomials in the lag operator of order p, i.e.

$$D(L) = \begin{bmatrix} \delta_{1,1}(L) & \dots & \delta_{1,n}(L) \\ \vdots & \ddots & \vdots \\ \delta_{n,1}(L) & \dots & \delta_{n,n}(L) \end{bmatrix}, \Phi(L) = \begin{bmatrix} \phi_{1,1}(L) & \dots & \phi_{1,r}(L) \\ \vdots & \ddots & \vdots \\ \phi_{r,1}(L) & \dots & \phi_{r,r}(L) \end{bmatrix}.$$

By substituting (2) into (1), the vector autoregressive form (FVAR) of the factor model can be written as

$$\begin{bmatrix} F_t \\ X_t \end{bmatrix} = \begin{bmatrix} \Phi(L) & 0 \\ \Lambda\Phi(L) & D(L) \end{bmatrix} \begin{bmatrix} F_{t-1} \\ X_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{F_t} \\ \varepsilon_{X_t} \end{bmatrix}, \tag{3}$$

where

$$\begin{bmatrix} \varepsilon_{F_t} \\ \varepsilon_{X_t} \end{bmatrix} = \begin{bmatrix} I \\ \Lambda \end{bmatrix} \eta_t + \begin{bmatrix} 0 \\ v_t \end{bmatrix},$$

with variance covariance matrix

$$E\varepsilon_t\varepsilon_t' = \Sigma_\varepsilon = \begin{bmatrix} \Sigma_\eta{}' & \Sigma_\eta{}'\Lambda' \\ \Lambda\Sigma_\eta{}' & \Lambda\Sigma_\eta{}'\Lambda' + \Sigma_v \end{bmatrix},$$

where $E\eta_t\eta_t' = \Sigma_\eta$ and $Ev_tv_t' = \Sigma_v$. Finally, inversion of the FVAR form yields the vector moving average form (VMA) for the X_t process

$$X_t = B(L)\eta_t + C(L)v_t,$$

where $B(L) = [I - D(L)L]^{-1}\,\Lambda\,[I - \Phi(L)L]^{-1}$ and $C(L) = [I - D(L)L]^{-1}$.

2.1 Estimation

The estimation problem may be written as follows

$$\min_{F_1,\ldots,F_T,\Lambda,D(L)} T^{-1}\sum_{t=1}^{T}\left[\left(I-D(L)L\right)X_t-\Lambda F_t\right]'\left[\left(I-D(L)L\right)X_t-\Lambda F_t\right],$$

and solved following an iterative process. Differently from Stock and Watson (2005), where the r static factors F_t are estimated as the first r principal components of $(I - D(L)L)\,X_t$, with r determined following information criteria, when a priori information concerning the economic interpretation of the factors is available, the estimation of the F_t factors can be carried out considering the relevant sub-set of variables. Therefore, given a preliminary estimate of $D(L)$, the r static factors F_t can be estimated as the first principal component of each of the r-subset of variables $(I_i - D_i(L)L)\,X_{i,t}$ $i = 1, \ldots, r$; then, conditional on the estimated static factors, an estimate of Λ and $D(L)$ can be obtained by OLS estimation of the block of equations corresponding to X_t in (1) . The procedure is then iterated until convergence. Once the final estimate of F_t is available, the $\Phi(L)$ matrix in (3) can be obtained by OLS estimation of the block of equations corresponding to F_t. Then, by also employing the final estimate of the Λ and $D(L)$ matrices, the restricted VAR coefficients in (3) can be computed. Stock and Watson (2005) provide details about the asymptotic properties, i.e. consistency and asymptotic normality, of the estimation procedure.

The availability of a priori information on the interpretation of the factors is key for the implementation of the proposed approach, which requires the separation of the variables in appropriate blocks. The separation issue is not problematic for econometric modelling, which is always guided by an underlying economic theory. In the application illustrated below a large-scale macroeconometric model for the G-7 countries (USA, Japan, euro area, UK, Canada) has been estimated, considering for each country eight macroeconomic variables, i.e. real output, inflation, short and long term nominal interest rates, nominal money balances, the real oil price, the real effective exchange rate and real stock prices. Hence, eight groups of homogeneous variables have been assessed and global factors bearing the interpretation of global GDP growth rate, real oil price growth, global real stock prices growth, and a global nominal factor, related to monetary policy management, have been extracted.

It should be finally noted that the proposed approach not only allows for a more straightforward interpretation of the factors, but also avoids contamination from series potentially unrelated to the phenomenon of interest, which could undermine the asymptotic theory justifying the use of principal components analysis. The latter assumes that the variability of the common component is not too small and that the cross-correlation in the idiosyncratic errors is not too large. If noise is added to the information set it can be expected that, as more variables are added to the data set, the average size of the common factors will decrease, while the correlation across idiosyncratic components will increase. Hence, beyond a certain threshold, increasing the cross-sectional dimension of the information set is not desirable any longer, and could also negatively affect the explanatory power of the model. See Boivin and Ng (2006) for additional details on this issue.

2.2 Identification of structural shocks

The identification of the structural shocks in the F-VAR model can be carried out as follows. Denoting by ξ_t the vector of the r structural global shocks, the relation between reduced form and structural form global shocks can be written as $\xi_t = H\eta_t$, where H is square and invertible. The identification of the structural shocks amounts then to the estimation of the elements of the H matrix. It is assumed that $E\left[\xi_t \xi_t'\right] = I_r$, and hence $H\Sigma_\eta H' = I_r$. Moreover, by denoting ψ_t the n structural idiosyncratic shocks, the relation between reduced form and structural form idiosyncratic shocks can be written as $\psi_t = \Theta v_t$, where Θ is square and invertible. The identification of the structural idiosyncratic shocks amounts then to the estimation of the elements of the Θ matrix. It is assumed that $E\left[\psi_t' \psi_t\right] = I_n$, and hence $\Theta \Sigma_v' \Theta = I_n$.

The VMA representation of the factor model in structural form can then be written as

$$X_t = B^*(L)\xi_t + C^*(L)\psi_t,$$

where $B^*(L) = B(L)H^{-1} = [I - D(L)L]^{-1} \Lambda [I - \Phi(L)L]^{-1} H^{-1}$, $C^*(L) = C(L)\Theta^{-1} = [I - D(L)L]^{-1} \Theta^{-1}$, and $E\left[\psi_{i,t}\xi_{j,t}\right] = 0$ any i, j. Given r factors, then $r(r-1)/2$ restrictions need to be imposed in order to exactly identify the structural global shocks. Moreover, exact identification of the n structural idiosyncratic shocks requires the imposition of additional $n(n-1)/2$ zero restrictions.

The imposition of the exactly identifying restrictions is easily achieved by following a double Cholesky strategy, guided by economic theory, carried out as follows. First, the structuralization of the factor or global shocks is achieved by assuming a lower triangular structure for the H matrix, with ordering of the variables set according to economic theory. For instance, standard economic assumptions concerning the speed of adjustment to shocks, i.e. slow (output, inflation), intermediate (interest rates, money growth), and fast (stock prices, exchange rates, commodity prices) variables, could be employed. The H matrix is then written as

$$H = \begin{bmatrix} h_{11} & & \\ \vdots & \ddots & \\ h_{r1} & \cdots & h_{rr} \end{bmatrix},$$

and estimated by the Choleski decomposition of $\hat{\Sigma}_\eta$, i.e. from $\xi_t = H\eta_t$ we have $E\left[\xi_t \xi_t^{'}\right] = H\Sigma_\eta H' = I$, and hence, $\hat{H}^{-1} = chol(\hat{\Sigma}_\eta)$. The identification scheme performed allows for exact identification of the r structural global shocks, imposing $r(r-1)/2$ zero restrictions on the contemporaneous impact matrix.

Second, matrix Θ is identified by imposing a lower triangular structure, with each non-zero block on the main diagonal showing a lower triangular structure as well, i.e.

$$\Theta = \begin{bmatrix} \Theta_{11} & & \\ \vdots & \ddots & \\ \Theta_{1r} & \cdots & \Theta_{rr} \end{bmatrix},$$

where

$$\Theta_{jj} = \begin{bmatrix} \theta_{jj,11} & & \\ \vdots & \ddots & \\ \theta_{jj,1m} & \cdots & \theta_{jj,mm} \end{bmatrix},$$

and $n = mr$, with m equal to the number of units in the sample, countries, for instance, as in the application provided. Again, economic theory is called for to guide the ordering of the different units in each block, which could be based, for instance, on the distinction in large and small countries, in terms of GDP.

The estimation of Θ is then carried out as follows:

(1) regress $\hat{\varepsilon}_{X,t}$ on $\hat{\xi}_t$ by OLS and obtain $\hat{v}_t$ as the residuals;
(2) then from $\psi_t = \Theta v_t$ we have $E\left[\psi_t \psi_t'\right] = \Theta \Sigma_v' \Theta = I$. Hence, $\hat{\Theta}^{-1} = chol(\hat{\Sigma}_v)$.

The identification scheme performed allows for exact identification of the n structural idiosyncratic shocks, imposing $n(n-1)/2$ zero restrictions on the contemporaneous impact matrix.

In the proposed application the double triangular structure has been justified on the basis of both the distinction in "slow" (output, inflation), "intermediate" (interest rates, money growth), and "fast"-moving variables (stock prices, exchange rates, commodity prices), and the distinction, in terms of GDP size, in large (USA, euro-12 area, Japan) and relatively small (UK, Canada) countries. Hence, for instance, the matrix Θ_{11} contains the GDP growth rate time series for the various countries in the following order: the US, Japan, the euro area, the UK and Canada.

Alternatively, in order to ensure robustness to variable ordering, estimation may be carried out by following the thick modelling estimation approach (Granger and Jeon 2004), consisting of repeating the analysis considering all the possible ordering of the variables, also simulating the model by Monte Carlo tools in each case, yielding median estimates and 95% confidence levels for the parameters of interests, i.e. impulse response functions and forecast error variance decomposition. Finally, policy analysis can also be carried out by means of generalized impulse response analysis (Pesaran and Shin 1998), which, by construction, is not affected by variables ordering. Therefore, estimation methods allowing to draw robust conclusions not only to the ordering of the variables, but also to potential misspecification of the econometric model, are available for the proposed approach.

3 Empirical application: the construction of a large-scale macroeconometric model for the G-7

Quarterly time series data for five countries, i.e. the US, Japan, the euro-12 area, the UK, and Canada, over the period 1980:1-2005:2, have been employed. Eight variables for each country have been considered, i.e. the real GDP growth rate, the real oil price growth rate, real stock market price returns, real effective exchange rate returns, the CPI inflation rate, the nominal money growth rate and the nominal short and long term interest rates. On the basis of misspecification tests, the lag length of the FVAR model has been set to one. On the whole, the econometric model is composed of 39 equations, with the first 35 equations referring to the 35 endogenous variables, i.e. real output growth, inflation, the nominal short term rate, the nominal long term rate, nominal money growth, real exchange rate returns, and real stock returns for the five countries in the system, and the last 4 equations referring to the global factors.

The proportion of total variance explained by the first factor/principal component for each group of variables is equal to 0.57 for real stock market returns, 0.40 for real output growth, 0.95 for real oil price growth and 0.65 for the nominal variables. In all cases the first principal component bears the interpretation of global factor since, according to the estimated factor loadings, all the corresponding variables react as expected. On the other hand, all the other principal components tend to capture idiosyncratic dynamics. Hence, the estimated factors have a clear-cut macroeconomic interpretation, being associated with global real output growth, global stock market dynamics, real oil price growth and global nominal/monetary developments, respectively.

The comparison with the standard Stock and Watson (2005) approach reveals that the first four principal components, extracted from the full set of 40 macroeconomic variables, jointly account for about 61% of total variance (34, 14, 8 and 6%, respectively), still bearing the same interpretation as the one found for the group analysis. In fact, in terms of proportion of explained variance for each series by each principal component, accounting on average for about 65 and 90% of the variance, it is possible to associate the first and second principal components with the nominal variables and the real oil price series, respectively. On the other hand, the third and fourth components can be associated with real stock prices and output, accounting on average for about 45 and 30% of the variance, respectively. By comparing the estimated factors, it can be concluded that the two approaches yield similar results, as in all the cases the correlation coefficient is never lower than 0.82 (0.99 for the oil price factor, 0.97 for the nominal factor, 0.89 for the output factor, and 0.82 for the stock return factor). Yet, in the light of the the higher proportion of explained variance, it can be concluded that the proposed methodology improves over the Stock–Watson approach also in terms of fit, allowing for a more accurate estimation of the factors.

The FVAR model has been estimated following the iterative procedure described in the methodological section, with 1,000 Monte Carlo simulations carried out for the implementation of the thick modelling approach. The BIC information criterion computed for the proposed model and the Stock–Watson model, yielding −74.45 and −74.36, respectively, provides additional support for the proposed methodology.

Selected cumulative median impulse response functions are reported in Fig. 1 for illustrative purposes. As shown in the plot, the response of real output to a unitary global output shock is similar for all countries, with the shock always leading to a permanent increase in real GDP. Similarly, the global nominal shock leads to a permanent impact on the price level, while the global stock market shock leads to a positive permanent impact on real stock prices for all countries. Yet, the magnitude of the long

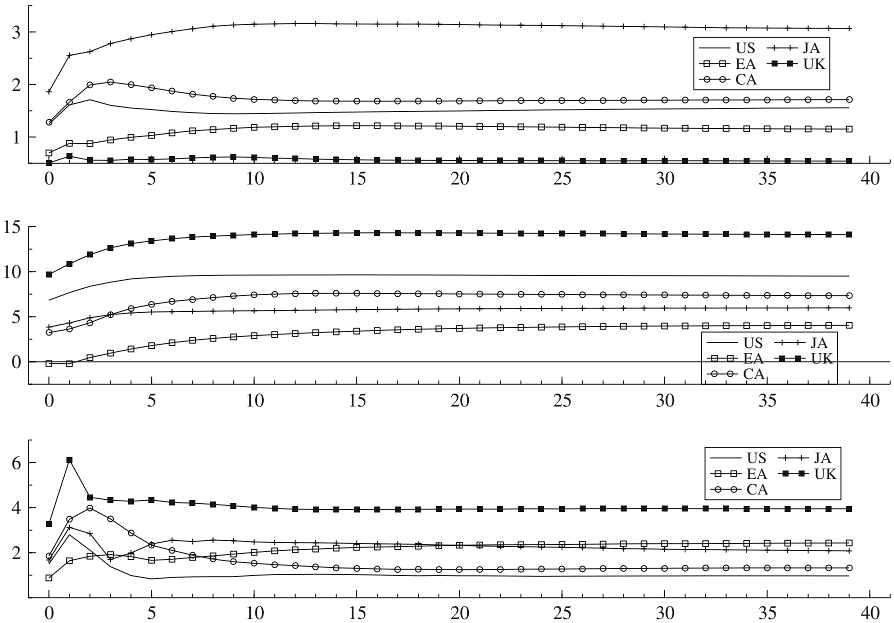

Fig. 1 Cumulative median impulse responses of real output (*top plot*), inflation (*center plot*), real stock prices (*bottom plot*) to positive global output, nominal and stock market shocks, respectively (*US* United States, *JA* Japan, *EA* euro area, *UK* United Kingdom, *CA* Canada)

term impact tends to vary across countries. For instance, for the global output shock the impact ranges between a minimum value of 0.54% (UK) and a maximum value of 3.07% (Japan), while the response for the other three countries is similar, averaging at 1.47%. A similar range of variation can also be noted for the long-term impact of the global stock market shock on real stock prices, spanning between a minimum of 0.97% (US) and a maximum of 3.95% (UK), while for the other three countries the impact averages at 1.93%. Finally, for the global nominal shock the range of variation is wider, i.e. 5.99 (Japan) to 14.11 (UK), while for the three remaining countries the average impact is 7.61%. A large number of issues can be explored by means of the estimated macromodel, ranging from the international transmission of global and idiosyncratic shock to international comovements in the real and nominal side of the economy. Applications along the above lines can be found in Bagliano and Morana (2006).

4 Conclusions

In this paper a new approach to factor vector autoregressive estimation, based on Stock and Watson (2005), is introduced. Relatively to the Stock–Watson approach, the proposed method has the advantage of allowing for a more clear-cut interpretation of the global factors, as well as for the identification of all idiosyncratic shocks. Moreover, as the Stock and Watson (2005) approach, the proposed methodology has the advantage of using an iterated procedure in estimation, recovering, asymptotically, full efficiency, and also allowing the imposition of appropriate restrictions concerning the lack of Granger causality of the variable versus the factors. Finally, relatively to other available methods, the modelling approach has the advantage of allowing for joint modelling of all the variables, with no need for long-run forcing hypotheses. An application to large scale macroeconometric modelling is also provided.

References

Bagliano FC, Morana C (2006) International macroeconomic dynamics: a factor vector autoregressive approach. Collegio Carlo Alberto Notebook Series, no. 32

Bai J (2002) Estimating cross-section common stochastic trends in nonstationary panel data. Boston College, Mimeo

Bai J (2003) Inferential theory for factor models of large dimensions. Econometrica 71:135–171

Bai J, Ng S (2004) A panic attack on unit roots and cointegration. Econometrica 72:1127–1177

Bernanke BS, Boivin J (2003) Monetary policy in a data-rich environment. J Monetary Econ 50:525–546

Bernanke BS, Boivin J, Eliasz P (2005) Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach. Quart J Econ 120:387–422

Boivin J, Ng S (2006) Are more data always better for factor analysis. J Econometrics 132:169–194

Favero CA, Marcellino M (2005) Large datasets, small models and monetary policy in Europe. CLM Econ, pp 249–269

Favero CA, Marcellino M, Neglia F (2005) Principal components at work: the empirical analysis of monetary policy with large data sets. J Appl Economet 20:603–620

Forni M, Hallin M, Lippi M, Reichlin L (2000) The generalized dynamic factor model: identification and estimation. Rev Econ Stat 82:540–554

Giannone D, Reichlin L, Sala L (2002) Tracking Greenspan: systematic and unsystematic monetary policy revisited. CEPR Discussion paper, no. 3550

Granger CW, Jeon Y (2004) Thick modelling. Econ Model 21:323–343

Pesaran MH, Shin Y (1998) Generalised impulse response analysis in linear multivariate models. Econ Lett 58:17–29

Pesaran MH, Schuerman T, Weiner SM (2004) Modelling regional interdependencies using a global error correcting macroeconometric model. J Bus Econ Stat 22:129–162

Stock JH, Watson MW (1998) Diffusion indexes, NBER Working Paper, no. 6702

Stock JH, Watson MW (2005) Implications of dynamic factor models for VAR analysis, NBER Working Paper, no. 11467