PhD Economics
Dynamic Macroeconomics

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Notes on: Job Matching and Unemployment Dynamics

General references:
Romer (2012) *Advanced Macroeconomics,* fourth editioon, chapter 10
Bagliano-Bertola (2007) *Models for dynamic macroeconomics*, chapter 5, sections 5.2-5.4

Specific references:
Pissarides (2000) *Equilibrium Unemployment Theory,* second edition, ch. 1-3
Petrongolo-Pissarides (2001) "Looking into the black box: a survey of the matching function", *Journal of Economic Literature*, June, 390-431
Nickell S., Nunziata L., Ochel W., Quintini G. (2002) "The Beveridge curve, unemployment and wages in the OECD from the 1960s to the 1990s", Centre for Economic Performance, LSE
Hornstein A., Krusell P., G.L. Violante (2005) "Unemployment and Vacancy fluctuations in the matching model: inspecting the mechanism", *Federal Reserve Bank of Richmond Economis Quarterly*
Shimer R. (2005) "The cyclical behavior of equilibrium unemployment and vacancies", *American Economic Review,* March
Hall R. (2005) "Employment fluctuations with equilibrium wage stickiness", *American Economic Review,* March

Motivation:

1. *theoretical*: unemployment viewed as a consequence of a "non-walrasian" labour market (equilibrium wage does not decrease when there are unemployed workers).
   Non-walrasian models:
   - efficiency wage models;
   - contracting models (implicit contracts, insider-outsider);
   - **search/matching models**, focused on heterogeneity of workers and firms; matching between (unemployed) workers and firms (with vacancies) is not ensured by the "market" but is achieved by a search process;
2. *empirical*: sizeable flows in and out of unemployment even at unchanged unemployment rate ("job creation" and "job destruction" processes).

Main features of matching models:

- matching on the labour market as result of a decentralized and uncoordinated process of search for workers and firms;
- basic economic mechanism: *search externality* $\rightarrow$ number of agents on the market affects the probability of matching of other agents (on both sides of the market).

Set-up:

- constant labour force L:

  labour supply : "employed" + "unemployed" $u\,L$ (searching for jobs)
  labour demand : "jobs" + "vacancies" $v\,L$ (posted to be filled)

  where u is the "unemployment rate" and v is the "vacancy rate";

- $u\,L$ and $v\,L$ are "inputs" of a *matching function* giving the number of successful matches between unemployed and vacancies in each instant of time (yielding "employed" workers and productive "jobs"):

$$m\,L = m\,(\underset{+}{u\,L}, \underset{+}{v\,L})$$

where m is the "matching rate".

With C.R.S. (to ensure constant unemployment rate in steady state):

$$m = \frac{m\,(u\,L, v\,L)}{L} \underset{\text{by C.R.S.}}{=} m\,(u, v)$$

- (istantaneous) probability of a match:
  - for an *unemployed worker*:

$$\frac{m\,(u, v)}{u} = m\left(1, \frac{v}{u}\right) \equiv p\,(\theta) \quad \text{with } p'\,(\theta) > 0$$

    where $\theta \equiv v/u$ is a measure of "*tightness*" of the labour market; the average duration of unemployment is $1/p(\theta)$;
  - for a *firm with a vacancy*:

$$\frac{m\,(u, v)}{v} = \frac{m\,(u, v)}{u}\frac{u}{v} = \frac{p\,(\theta)}{\theta} \equiv q\,(\theta) \quad \text{with } q'\,(\theta) < 0$$

    and the average duration of a vacancy is $1/q(\theta)$;

$\Rightarrow$ *externalities*: dependence of probabilities p and q on ratio v/u.

Unemployment dynamics

With exogenous (istantaneous) "separation rate" s determining inflows and successful matches determining outflows, the dynamics of the unemployment rate is:

$$\dot{u}\,L = \underbrace{s\,\underbrace{(1-u)\,L}_{\text{employed}}}_{\text{inflow}} - \underbrace{p(\theta)\,\underbrace{u\,L}_{\text{unempl.}}}_{\text{outflow}}$$

$$\Rightarrow \quad \dot{u} = s\,(1-u) - p(\theta)\,u$$

Stationary relation between u and θ (and also between v and u: "Beveridge curve"):

$$\dot{u} = 0 \;\Rightarrow\; u = \frac{s}{s + p(\theta)} \quad \text{with} \quad \left.\frac{d\theta}{du}\right|_{\dot{u}=0} < 0$$

Shifts of the curve due to changes in s and properties of $m(.)$ ($\rightarrow p(.)$), capturing the efficiency of the matching process on the labour market.
Dynamics:

$$\frac{d\dot{u}}{du} < 0$$

Supply of vacancies

Each firm has 1 job: when occupied, it yields an istantaneous output flow of y and entails labour costs w; if it is not occupied and the firm posts the vacancy on the market, there is an istantaneous search cost c.

Vacancies are opened only if they are "profitable" for firms. In solving its infinite-horizon, (expected) profit-maximization problem, each individual firm takes aggregate conditions on labour market (i.e. tightness θ) as *given*.

$$\Rightarrow \text{ optimal choice: open a vacancy until its value } V > 0$$

Assuming free entry of firms on the market, profit maximization ensures that $V(t) = 0$ for all t.

Value of a vacancy V and of a filled job J. The values for the firm of both "assets" (vacancy and job) can be expressed by means of similar dynamic equations:

$$r\,V(t) = -c + q(\theta(t))\;(J(t) - V(t)) + \dot{V}(t)$$

$$r\,J(t) = (y - w(t)) + s\;(V(t) - J(t)) + \dot{J}(t)$$

Stationary equilibrium: $\dot{V}(t) = \dot{J}(t) = 0$ and by profit maximization $V = 0$.

$$\left.\begin{array}{l} J = \frac{c}{q(\theta)} \\[2ex] J = \frac{y-w}{r+s} \end{array}\right\} \Rightarrow \underbrace{\underbrace{y - w}_{\text{net marg. prod.}} = \underbrace{(r+s)\,\frac{c}{q(\theta)}}_{\text{"adjustment costs"}}}_{\text{"job creation condition"}}$$

Wage determination

Simplifying assumption: wage w is (exogenously) fixed at a constant level $\bar{w} < y$.

(Alternative assumption: w is endogenously determined by a decentralized bargaining process for each firm-worker pair after a match $\rightarrow$ *Nash bargaining*)

Steady state

Three equations describe the economy in steady state equilibrium:

$$u = \frac{s}{s + p(\theta)} \qquad \text{(Beveridge curve, BC)}$$

$$y - w = (r + s)\,\frac{c}{q(\theta)} \qquad \text{(job creation condition, JC)}$$

$$w = \bar{w} \qquad \text{(fixed wage, W)}$$

Recursive structure: $(JC + W) \Rightarrow w, \theta \Rightarrow (BC)\ u, v.$

Application: effect of *aggregate* (y) versus *sectoral* (s) disturbances on u and v.

Dynamics

Outside the steady state, changes in the aggregate labour market conditions θ affect the unemployment rate dynamics according to:

$$\dot{u}(t) = s\,(1 - u(t)) - p(\theta(t))\,u(t)$$

but dynamics of θ results from unemployment dynamics and firms' behaviour.

Dynamics of θ.
Recall that firms' profit maximization yields $V(t) = 0$ for all t $\quad\Rightarrow\quad$ $\dot{V}(t) = 0$ for all t.

$$\Rightarrow J(t) = \frac{c}{q(\theta(t))} \quad \text{valid for all } t$$

Outside the steady st. J changes according to its dynamic equation with $V(t) = 0$:

$$r\,J(t) \;=\; (y - \bar{w}) - s\,J(t) + \dot{J}(t)$$

$$\Rightarrow \quad J(t) = \int_t^{\infty} (y - \bar{w})\,\mathrm{e}^{-(r+s)\tau} d\tau$$

Dynamics of θ is finally derived (assuming constant elasticity of $p(\theta)$: $p'\theta/p \equiv \eta$):

$$J(t) = \frac{c}{q(\theta(t))} \quad \Rightarrow \quad \dot{J}(t) = \frac{c}{p(\theta(t))}\,(1-\eta)\,\dot{\theta}(t)$$

using dynamic equation for J and again $J = c\theta/p(\theta)$:

$$\dot{\theta}(t)\,\frac{c}{p(\theta(t))}\,(1-\eta) = (r+s)\,\frac{c\theta(t)}{p(\theta(t))} - (y - \bar{w})$$

$$\Rightarrow \dot{\theta}(t) = \frac{r+s}{1-\eta}\,\theta(t) - \frac{p(\theta(t))}{c(1-\eta)}\,(y-\bar{w})$$

$\Rightarrow \dot{\theta}$ depends only on θ with no independent role for u

- $\dot{\theta} = 0 \Rightarrow$ horizontal line in space (θ, u) at steady state value $\bar{\theta}$
- firms' decisions on vacancies to open makes dynamics of θ outside the steady state "unstable":

$$\left.\frac{\partial \dot{\theta}}{\partial \theta}\right|_{\dot{\theta}=0} > 0$$

Two-equation dynamic system linearized around steady state values of u and θ ($\bar{u}$ and $\bar{\theta}$):

$$\begin{pmatrix} \dot{u} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \overset{(-)}{-(s+p(\bar{\theta}))} & \overset{(-)}{-\bar{u}p'(\bar{\theta})} \\ 0 & \underset{(+)}{r+s} \end{pmatrix} \begin{pmatrix} u - \bar{u} \\ \theta - \bar{\theta} \end{pmatrix}$$

$\Rightarrow$ determinant < 0 : saddlepoint equilibrium.

Problems

1. Using the steady state equations for the (u, θ) relation (BC), the job creation condition (JC) and assuming a fixed wage rate $\bar{w}$,compare the effects on the steady state levels of u, v and θ of a smaller labour productivity ($\Delta y < 0$, an *aggregate* shock) and of a higher separation rate ($\Delta s > 0$, a *sectoral* shock). [Formal derivation is not required; graphical analysis with economic explanation is sufficient]

2. Using the two dynamic equations for the unemployment rate u and the degree of tightness of the labour market θ, derive the effect on the steady state of the economy of an *anticipated* future increase in the wage rate $\bar{w}$ (firms know at time t_0 that the wage rate will increase permanently at a future date t_1). Describe the dynamic adjustment of u, v and θ towards the new steady state equilibrium.